Heterotic Orbifold Models

杂化轨道模型

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Contents

目录

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Abstract

摘要

We review efforts in string model building, focusing on the heterotic orbifold compactifications. We survey how one can, starting from an explicit string theory, obtain models which resemble Nature. These models exhibit the standard model gauge group, three generations of standard model matter, and an appropriate Higgs sector. Unlike many unified models, these models do not suffer from problems such as doublet-triplet splitting, too rapid proton decay, and the μ problem. Realistic patterns of fermion masses emerge, which are partly explained by flavor symmetries, including their modular variants. We comment on challenges and open questions.

我们回顾弦论模型构建的相关研究，重点聚焦杂化 orbifold 紧致化。我们概述了如何从一个明确的弦论出发，得到近似真实自然的模型。这些模型具备标准模型规范群、三代标准模型物质以及合适的希格斯区。与许多大统一模型不同，这类模型不存在双三角分裂、质子衰变速率过快以及 μ 问题。它们可以得到符合实际的费米子质量模式，部分质量模式可由味对称性 (包括其模形式变体) 解释。我们还对相关挑战与开放问题进行了评述。

Keywords

关键词

String compactifications · Heterotic string · Model building ·

弦紧致化 · 杂弦 · 模型构建 ·

Phenomenology · Symmetries of string models

唯象学 · 弦模型的对称性

Introduction

引言

If string theory is to describe the real world, it has to reproduce our current established understanding of physics. In particular, its low-energy description has to give rise to the standard model (SM). Generally, string model building concerns the question of how the SM fits into string theory. In practice, one compactifies a consistent string theory to a four-dimensional model which can be studied and confronted with observation. One particularly important aspect of top-down model building is that the globally consistent models are complete, that is, they do not only describe the SM but also include, say, the degree(s) of freedom driving cosmic inflation and dark matter. That is, unlike in the bottom-up approach, one cannot add extra sectors to the model at will.

如果弦理论要描述真实世界，它就必须复现我们目前已确立的物理学认知。具体而言，它的低能描述必须能够得出标准模型 (SM)。一般来说，弦模型构建关注的是标准模型如何嵌入弦理论的问题。在实际操作中，研究者会将自洽的弦理论紧致化为四维模型，进而开展研究并和观测结果比对。自上而下模型构建的一个尤其重要的特点是，全局自洽的模型是完备的: 也就是说，它们不仅能描述标准模型，还包含了 (比如) 驱动宇宙暴胀和解释暗物质的自由度。这意味着，和自下而上的方法不同，研究者无法随意向模型中添加额外 sectors。

Historically, the first attempts to construct realistic string models were based on the heterotic string. It was noticed that the structure of SM is remarkably consistent with unification along the exceptional chain $\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6 \subset \mathrm{E}_7 \subset \mathrm{E}_8$ [75]. In this chapter we provide a brief overview of orbifold compactifications of the heterotic string which come close to the SM.

从历史上看，构建现实弦模型的最早尝试是基于杂化弦的。研究者早就注意到，标准模型的结构与例外群链 $\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6 \subset \mathrm{E}_7 \subset \mathrm{E}_8$ [75] 的统一理论惊人地相容。本章我们简要概述接近标准模型的杂化弦 orbifold 紧致化。

Heterotic models come broadly in two classes, they can either be based on smooth compactifications [24] or on orbifolds [28, 29], cf. Fig. 1. These classes are related as some smooth compactifications can emerge from orbifolds via blowup (cf. e.g., [13]). Orbifolds have the advantage that their construction involves explicit strings, which is why they will be our focus. Orbifolds can be constructed in the so-called free fermionic approach, yet our focus will be the classical approach, in which the so-called symmetric orbifolds have a geometric interpretation.

杂化模型大致分为两类: 一类基于光滑紧致化 [24]，另一类基于 orbifold[28, 29]，参见图 1。这两类是相关联的: 部分光滑紧致化可以通过 orbifold 吹胀得到 (参见例如 [13])。Orbifold 的优势在于其构造涉及显式弦，这也是我们将它作为本章研究重点的原因。Orbifold 可以通过所谓的自由费米方法构造，但我们关注的是经典方法，其中的对称 orbifold 具有几何诠释。

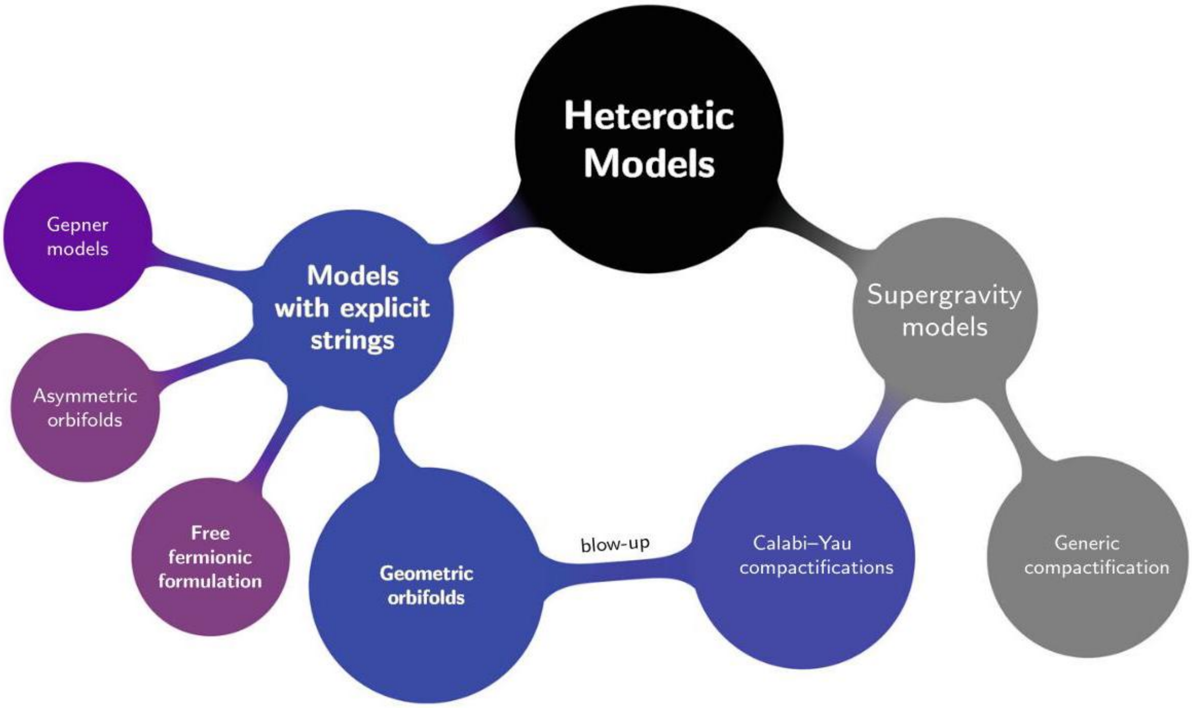

Fig. 1 An incomplete survey of heterotic models. The focus of this chapter is on the constructions that are typeset bold

图 1 杂化模型不完全概览。本章重点关注加粗排印的构造

The purpose of this chapter is to summarize the current status of heterotic model building. There are already excellent reviews of this subject such as [57]; however, our focus will be on more recent developments and a clear account of the open questions. To this end, we will review the target of string model building, the SM, and some of its extensions in section "What Do We (Believe to) Know?" before turning to the heterotic string and its compactifications in section "Compactifying the Heterotic String." In sections "Spectrum" and "Symmetries of the Effective Action" we collect some facts about the spectra and symmetries of the constructions, which will be the basis for the discussion of challenges in section "Challenges." In section "Examples" we provide some explicit examples. After briefly commenting on smooth heterotic compactifications in section "Smooth Compactifications," we provide an outlook in section "Where Do We Stand?".

本章的目的是总结杂化弦模型构建的研究现状。关于这一课题已经有诸多出色的综述，例如 [57]；但我们将重点关注更近的研究进展，并清晰梳理待解决的开放问题。为此，我们会先在「我们 (相信) 知道什么?」一节回顾弦模型构建的目标——标准模型，以及它的部分延伸，之后再在「紧致化杂化弦」一节讨论杂化弦及其紧致化。我们会在「谱」与「有效作用量的对称性」两节整理这类构造的谱与对称性相关结论，以此作为「挑战」一节讨论相关问题的基础。我们会在「例子」一节给出若干显式例子。在「光滑紧致化」一节简要讨论杂化弦光滑紧致化后，我们会在「我们目前进展如何?」一节给出展望。

What Do We (Believe to) Know?

我们 (自认) 知道什么？

Before delving into what string theory gives us, let us briefly survey what we expect to get out of string model building.

在深入探讨弦理论给出的结论之前，我们先简要梳理一下我们对弦模型构建期望得到的结果。

A Very Short Recap of the SM

标准模型简述

First and foremost, we wish to obtain a quantum field theory (QFT) that is consistent with the SM (see, e.g., [63] for a detailed description). The latter is based on the continuous gauge symmetry (Strictly speaking we do not really know the gauge group of the SM but only its Lie algebra, a subtlety which we will, like most of the literature, not discuss in detail.)

首先，我们希望得到一个与标准模型 (SM) 相容的量子场论 (QFT)(详见文献 [63])。标准模型建立在连续规范对称性之上 (严格来说，我们并未真正知晓标准模型的规范群，仅知道它的李代数。和大多数文献一样，我们不会对此细节展开详细讨论。)

$$G_{\mathrm{SM}} = \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}. \tag{1}$$

The matter content consists of three generations of quarks and leptons, left-chiral Weyl fermions which transform as

物质内容包含三代夸克与轻子，即左手外尔费米子，它们的变换满足

$$\text{quarks}: q_f = (\mathbf{3},\mathbf{2})_{1/6}, \bar{u}_f = \left(\bar{\mathbf{3}},\mathbf{1}\right)_{-2/3}, \bar{d}_f = \left(\bar{\mathbf{3}},\mathbf{1}\right)_{1/3}, \tag{2a}$$

$$\text{leptons}: \ell_f = (\mathbf{1},\mathbf{2})_{-1/2}, \bar{e}_f = (\mathbf{1},\mathbf{1})_1 \tag{2b}$$

under G_{SM}. Here, $f \in \{1,2,3\}$ labels the generations. In addition, there is the so-called Higgs field, a complex scalar carrying the quantum numbers $h = (\mathbf{1},\mathbf{2})_{1/2}$. The Higgs acquires a vacuum expectation value (VEV), $\langle h \rangle \sim 100\mathrm{GeV}$, which breaks G_{SM} down to $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{em}}$, under which (2) are vector-like and acquire masses which are given by the product of so-called Yukawa couplings $Y_{u,d,e}$ and $\langle h \rangle$. In the SM, the Yukawa couplings are input parameters which are adjusted to fit data. A curious fact about the SM is that the combination of charge conjugation and parity, $\mathcal{CP}$, is broken in the flavor sector, i.e., by the Yukawa couplings, but seemingly not in the strong interactions. This mismatch gets referred to as the strong $\mathcal{CP}$ problem. The neutrinos, which are part of the ℓ_f, are also massive, yet it is currently not known which operator describes their mass. The most plausible options are the Weinberg operator, $\kappa_{gf}\,(\ell^g h)\left(\ell^f h\right)$, or Dirac neutrino masses, in which case one has to amend (2) by right-handed neutrinos υ^f . The neutrino masses are much smaller than the masses of the charged fermions.

在 G_{SM} 下。此处 $f \in \{1,2,3\}$ 标记代。此外还有所谓希格斯场，它是一个带有量子数 $h = (\mathbf{1},\mathbf{2})_{1/2}$ 的复标量。希格斯获得真空期望值 (VEV) $\langle h \rangle \sim 100\mathrm{GeV}$ ，将 G_{SM} 破缺为 $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{em}}$ ，在此下式 (2) 中的粒子是矢量型的，并获得质量，质量由汤川耦合 $Y_{u,d,e}$ 与 $\langle h \rangle$ 的乘积给出。在标准模型中，汤川耦合是拟合实验数据的输入参数。标准模型一个值得注意的特点是，电荷共轭与宇称的组合 $\mathcal{CP}$ 在味道部分被汤川耦合破缺，但在强相互作用中似乎没有被破缺。这种矛盾被称为强 $\mathcal{CP}$ 问题。中微子作为 ℓ_f 的一部分也具有质量，但目前人们仍不清楚描述其质量的算符是什么。最合理的选项是温伯格算符 $\kappa_{gf}\left(\ell^g h\right)\left(\ell^f h\right)$ ，或是狄拉克中微子质量，若为后者则需要在式 (2) 中补充右手中微子 υ^f 。中微子质量远小于带电费米子的质量。

It is instructive to survey the continuous parameters of the SM. They comprise (i) 3 gauge couplings; (ii) θ_{QCD} ; (iii) 2 Higgs parameters; (iv) 12 masses; (v) 8 or 10 mixing parameters, depending on whether neutrinos are Dirac or Majorana particles. In a stringy completion, these parameters should be predicted rather than adjusted, and, as we shall discuss in section ”Challenges,” the requirement to reproduce these observables remains one of the greatest challenges in string model building. Note also that currently the only other parameters that we need to describe observation are the Planck mass M_{P} (or, equivalently, G_{Newton}), the vacuum energy ρ_{vacuum} , and the density to dark matter, ρ_{DM} . That is, currently the bulk of the (ununderstood) parameters of Nature resides in the flavor sector of the SM.

梳理标准模型的连续参数很有意义: 这些参数包括 (i) 3 个规范耦合常数；(ii) θ_{QCD} ；(iii) 2 个希格斯参数；(iv) 12 个质量；(v) 8 或 10 个混合参数，具体数量取决于中微子是狄拉克还是马约拉纳粒子。在弦论完备化中，这些参数应当是可以预言的，而非人为拟合的。正如我们在“挑战”一节将要讨论的，重现这些可观测物理仍然是弦模型构建中最大的挑战之一。另外还要注意，目前描述观测结果所需的其他参数只有普朗克质量 M_{P} (或等价的 G_{Newton})、真空能 ρ_{vacuum} ，以及暗物质密度 ρ_{DM} 。也就是说，目前自然界中大多数 (尚未被理解的) 参数都集中在标准模型的味道部分。

An important fact about the SM is that it does not only provide us with couplings and interactions that have been confirmed in experiments, but it also comes with tight constraints on additional particles and interactions. In particular, it is extremely hard to make extra states which are chiral w.r.t. G_{SM} consistent with observation. Also, while the Weinberg operator is a nonrenormalizable operator that is ”good” in the sense that it can describe neutrino masses, other higher-dimensional operators are highly constrained. For instance, the suppression scale of the dimension-6 operators leading to proton decay has to exceed $10^{15}\mathrm{GeV}$.

标准模型的一个重要特点是，它不仅给出了经实验验证的耦合与相互作用，还对额外粒子与相互作用给出了严格约束。具体而言，引入相对于 G_{SM} 手征的额外态并让它符合观测是极其困难的。此外，尽管温伯格算符作为不可重整算符可以很好地描述中微子质量，但其他高维算符受到严格约束。例如，导致质子衰变的 6 维算符的压低标度必须大于 $10^{15}\mathrm{GeV}$ 。

Early Universe

早期宇宙

The early universe provides us with important insights into high energy physics (see, e.g., [9]). For instance, big bang nucleosynthesis (BBN) works very well within the SM, and extra particles may be inconsistent with the primordial formation of the elements if they decay late or increase the Hubble expansion rate too much. In addition, fractionally charged particles are often stable since they cannot decay into SM states, and there are stringent constraints on their relic abundance (cf. e.g., [51, 67]).

> 早期宇宙为我们提供了高能物理的重要洞见 (参见例如 [9])。例如，大爆炸核合成 (BBN) 在标准模型 (SM) 框架下运转得非常好，如果额外粒子发生晚期衰变，或是过度增大哈勃膨胀率，那么这类粒子就可能与元素的原初形成观测结果不一致。此外，分数电荷粒子由于无法衰变为标准模型 (SM) 粒子态，通常保持稳定，因此它们的残余丰度受到严格约束 (参见例如 [51, 67])。

However, the early universe also requires physics beyond the SM. Most notably we need a field or sector that drive inflation, or another ingredient which provides us with solutions to the so-called horizon and flatness problems. In addition, there is very compelling evidence for dark matter which cannot be made of SM particles. Furthermore, the baryon asymmetry of the universe requires physics beyond the SM, too.

> 然而，早期宇宙也要求存在超出标准模型 (SM) 的新物理。最值得注意的是，我们需要一个场或能区来驱动暴胀，或是引入其他成分来解决所谓的视界问题与平坦性问题。此外，目前已有极具说服力的证据表明暗物质无法由标准模型 (SM) 粒子构成。更进一步，宇宙的重子不对称性也同样要求超出标准模型 (SM) 的新物理。

Beyond the Standard Model (BSM) Scenarios

超出标准模型 (BSM) 的物理场景

Having seen that physics beyond the SM is required to accommodate astrophysics and cosmology, let us spend some words on beyond the standard model (BSM) scenarios.

> 既然我们已经知道，为解释天体物理与宇宙学观测，必须存在超出标准模型 (SM) 的新物理，下面我们来介绍超出标准模型 (BSM) 的各类场景。

The supersymmetric variants of the SM (see, e.g., [68] for an introduction), most notably the minimal supersymmetric standard model (MSSM), have received substantial attention in the past decades. This is because, assuming low-energy supersymmetry (SUSY), the electroweak scale gets stabilized against quantum corrections. Of course, given the absence of clear signals for SUSY at the Large Hadron Collider (LHC), this scheme has lost some of its popularity in the recent years, yet the MSSM is arguably still one of the best motivated and well-defined BSM scenarios. The MSSM has a number of shortcomings which one may hope to solve in ultraviolet (UV) completions, and the purpose of this chapter is to discuss these solutions in section "Examples." In order to understand some of these shortcomings, let us look at the G_{SM} invariant superpotential terms up to order 4, SM 的超对称变体 (例如，文献 [68] 可作为入门介绍)，其中最受关注的就是最小超对称标准模型 (MSSM)，在过去几十年受到了广泛关注。这是因为，假设存在低能超对称 (SUSY)，电弱能标可以稳定，不受量子修正的影响。当然，由于大型强子对撞机 (LHC) 至今未发现清晰的 SUSY 信号，近年来这套理论的热度有所下降，但 MSSM 仍然可以说是动机最充分、定义最完善的 BSM 场景之一。MSSM 本身存在很多缺陷，人们希望可以在紫外 (UV) 完备理论中解决这些问题，本章的目的就是在“实例”一节讨论这些解决方案。为了说明这些缺陷，我们来看到 4 阶为止的 G_{SM} 不变超势项，

$$\begin{aligned}\mathcal{W}_{\text{gauge invariant}} &= \mu h_d h_u + \kappa_i \ell_i h_u \\ &+ Y_e^{gf} \ell_g h_d \bar{e}_f + Y_d^{gf} q_g h_d \bar{d}_f + Y_u^{gf} q_g h_u \bar{u}_f \\ &+ \lambda_{gfk} \ell_g \ell_f \bar{e}_k + \lambda'_{gfk} \ell_g q_f \bar{d}_k + \lambda''_{gfk} \bar{u}_g \bar{d}_f \bar{d}_k \\ &+ \kappa_{gf} h_u \ell_g h_u \ell_f + \kappa^{(1)}_{gfk\ell} q_g q_f q_k \ell_\ell + \kappa^{(2)}_{gfk\ell} \bar{u}_g \bar{u}_f \bar{d}_k \bar{e}_\ell, \end{aligned} \tag{3}$$

where we, in a slight abuse of notation, denoted the superfields by the same symbols as the SM fields in Equation (2). Note also that the MSSM has two Higgs doublets, h_u and h_d . The couplings Y_u, Y_d , and Y_e are the Yukawa couplings which yield the masses of quarks and charged leptons. The R -parity violating terms κ_i, λ_{gfk} , λ'_{gfk} , and λ''_{gfk} have to be highly suppressed and get often forbidden by R (or matter) parity, the origin of which is to be clarified in a UV completion of the model. The μ term in the first line of Equation (3) can a priori have any size, but in order to have proper electroweak symmetry breaking and sufficiently heavy Higgsinos it should be of the order TeV or so, which is a common choice for the soft SUSY breaking masses. Explanations of this fact comprise the Kim-Nilles [58] and Giudice-Masiero [45] mechanisms, and we will see later in section ”Examples” both are realized in explicit stringy completions of the MSSM. Further, while the κ term in the last line of Equation (3) can describe neutrino masses, the $\kappa^{(i)}$ terms have to be very small, e.g., the coefficients $\kappa^{(1)}_{1121}$ and $\kappa^{(1)}_{1122}$ have to be suppressed by more than $10^8 \cdot M_{\mathrm{P}}$, where $\sqrt{8\pi} M_{\mathrm{P}} = M_{\text{Planck}} \simeq 1.2 \cdot 10^{19}\,\mathrm{GeV}$. A proper understanding of this suppression arguably requires a solution within a consistent theory of quantum gravity, such as the explicit string models we consider here.

此处我们稍微滥用记号，将超场用式 (2) 中 SM 场的相同符号表示。还需要注意，MSSM 包含两个希格斯二重态，h_u 和 h_d 。耦合 Y_u, Y_d 和 Y_e 是汤川耦合，对应夸克和带电轻子的质量。破坏 R 宇称的项 κ_i, λ_{gfk} 、λ'_{gfk} 和 λ''_{gfk} 必须受到极强的压低，通常会被 R 宇称 (或物质宇称) 禁戒，该宇称的起源需要在该模型的紫外完备理论中得到说明。式 (3) 第一行的 μ 项原则上可以取任意大小，但为了实现正确的电弱对称性破缺，并得到足够重的希格斯微子，该项的量级需要在 TeV 左右，这也是软超对称破缺质量的常用选择。对这一性质的解释包括 Kim-Nilles 机制 [58] 和 Giudice-Masiero 机制 [45]，我们会在后文“实例”一节看到，二者都可以在 MSSM 的显式弦论完备化中实现。此外，式 (3) 最后一行的 κ 项可以描述中微子质量，而 $\kappa^{(i)}$ 项必须非常小，例如系数 $\kappa^{(1)}_{1121}$ 和 $\kappa^{(1)}_{1122}$ 需要被压低超过 $10^8 \cdot M_{\mathrm{P}}$ ，其中 $\sqrt{8\pi} M_{\mathrm{P}} = M_{\text{Planck}} \simeq 1.2 \cdot 10^{19}\,\mathrm{GeV}$ 。要恰当理解这种压低，需要在自洽量子引力理论中给出解决方案，例如此处我们讨论的显式弦模型。

Another appealing feature of the MSSM with low-energy SUSY is that gauge couplings unify remarkably well at a scale $M_{\mathrm{GUT}} \simeq$ few $\times 10^{16}\,\mathrm{GeV}$ [34]. This has led to the scheme of SUSY Grand Unified Theorys (GUTs) (see, e.g., [83] for an extended discussion), in which a unified symmetry like SU(5) or SO(10) gets broken at

M_{GUT} down to G_{SM} . An arguably even stronger motivation for GUTs is the structure of matter since one generation of the SM (cf. Equation 2) including the right-handed neutrino its into a 16-plet of SO(10),

带低能 SUSY 的 MSSM 的另一吸引人的性质是，规范耦合会在能标 $M_{\text{GUT}} \simeq$ (几 $\times 10^{16}$GeV) 处 remarkably well 统一 [34]。这催生了超对称大统一理论 (GUT) 框架 (拓展讨论可见例如 [83])，在这类理论中，统一对称性如 SU(5) 或 SO(10) 会在 M_{GUT} 处破缺为 G_{SM} 。可以说，GUT 更强的动机来自物质结构:SM 的一代物质 (见式 (2))，加上右手中微子，恰好可以填入 SO(10) 的 16 维表示，

$$\mathbf{16} = \underbrace{(\mathbf{3},\mathbf{2})_{1/6} + \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-2/3} + (\mathbf{1},\mathbf{1})_{1}}_{=\mathbf{10}} + \underbrace{\left(\overline{\mathbf{3}},\mathbf{1}\right)_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2}}_{=\overline{\mathbf{5}}} + (\mathbf{1},\mathbf{1})_{0}, \tag{4}$$

where we also indicated the SU(5) representations in underbraces. While the GUT symmetries work very well for the matter, they fail for the SM Higgs. The smallest SU(5) representations that contain the MSSM Higgs doublets are $\mathbf{5} + \overline{\mathbf{5}}$, which combine to a 10-plet of SO (10) . The additional $\text{SU}(3)_C$ 3-plets contained in these representations pose major threats to the model as they typically mediate unacceptably large proton decay unless their mass exceeds the Planck scale. This is one facet of the doublet-triplet splitting problems which haunt GUTs in 4D. On the other hand, as has been pointed out early on in the context of string model building, in higher-dimensional, in particular stringy, models the same mechanism that breaks the GUT symmetry can also split the doublets from the triplets [19,24].

我们在下划线中也标出了 SU(5) 表示。大统一理论 (GUT) 对称性对物质场的拟合效果非常好，但对标准模型 (SM) 希格斯场却不成立。包含最小超对称标准模型 (MSSM) 希格斯二重态的最小 SU(5) 表示为 $\mathbf{5} + \overline{\mathbf{5}}$ ，它们组合形成了 SO (10) 的 10 重态。这些表示中额外包含的 $\text{SU}(3)_C$ 3 重态给模型带来了重大问题: 除非它们的质量超过普朗克尺度，否则通常会引发不可接受的大规模质子衰变。这就是困扰四维大统一理论的二重态-三重态分裂问题的一个方面。另一方面，早在弦模型构建的早期就有研究指出，在高维 (尤其是弦论框架下的) 模型中，打破 GUT 对称性的同一机制也可以将二重态与三重态分离开 [19,24]。

What Do We Hope to Learn from String Model Building?

我们希望从弦模型构建中学到什么?

Of course, it will be reassuring to find a compactification which reproduces the SM in great detail, regardless of whether or not the underlying construction is unique. What is more, given such a model, we will be in a unique position to answer some of the most popular questions of our time:

当然，找到一个能高度精准重现标准模型 (SM) 的紧致化方案无疑会令人安心，无论基础构造是否唯一。更重要的是，拥有这样一个模型后，我们就能占据独特优势，解答当代最受关注的若干问题:

1. What is the origin of flavor and $\mathcal{CP}$ violation?

1. 味和 $\mathcal{CP}$ 破坏的起源是什么?

2. What is the nature of dark matter and what are the properties of the dark (aka hidden) sector?

2. 暗物质的本质是什么？暗 (即隐藏) sector 有哪些性质？

3. What drives cosmic inflation?

3. 是什么驱动了宇宙暴胀？

While these questions might be answered separately, the power of addressing them in explicit string models is that the answers are much more specific and related in intriguing ways.

尽管这些问题或许可以分开解答，但在明确的弦模型中研究它们的优势在于，得到的答案具体得多，且彼此间存在奇妙的关联。

Compactifying the Heterotic String

杂弦紧致化

Heterotic String

杂弦

In this section, we collect some basic facts on the heterotic string. For further details and a broader overview see Chap. 47, ”A Lightning Introduction to String Theory”. The term ”heterotic” derives from the Greek word ”hetero,” which translates as ”other,” and in biology is related to ”vigorous hybrid,” which arguably reflects the nature of the heterotic string. The heterotic string theory [43] is the result of combining a 10D superstring and a 26D bosonic string. The former can equip the theory with $\mathcal{N} = 1$ supersymmetry in ten dimensions whereas the bosonic string provides us with a non-Abelian gauge group of rank 16 (The nonsupersymmetric heterotic string can be obtained from this version, as we briefly describe in section ”Geometric Orbifolds Without SUSY.”),

本节我们整理杂弦的一些基本事实。更多细节与更全面的概述可参见第 47 章“弦论极速导论”。“杂 (heterotic)”一词源自希腊语“hetero”，意为“不同的”，在生物学中与“活性杂种”相关，可以说这一名称恰好体现了杂弦的本质。杂弦理论 [43] 是 10 维超弦与 26 维玻色弦结合的产物。前者为该理论赋予十维下 $\mathcal{N} = 1$ 超对称，而玻色弦则给出秩为 16 的非阿贝尔规范群 (我们会在“无超对称几何轨道面”一节简要说明，非超对称杂弦可由这个版本得到)，

$$G_{\text{het}} = \mathrm{E}_8 \times \mathrm{E}_8 \text{ or } \mathrm{SO}(32) . \tag{5}$$

Note that the most general compactification can have continuous enhancements of these gauge symmetries, yet we will mainly focus on (5). The heterotic theories contain only oriented closed strings propagating in ten dimensions.

注意，最一般的紧化可以让这些规范对称性发生连续增强，但我们主要聚焦于式 (5)。杂化理论仅包含在十维中传播的定向闭弦。

In light cone gauge, there are 8 right-moving bosonic string coordinates $X_{\mathrm{R}}^{i}(t-\sigma)$ and 8 right-moving fermions $\psi_{\mathrm{R}}^{i}(t-\sigma)$, where $2 \leq i \leq 9$.t and σ denote the world-sheet coordinates. There are in total 24 left-moving coordinates $X_{\mathrm{L}}^{M}(t+\sigma)$. In symmetric compactifications they get decomposed into $X_{\mathrm{L}}^{i}(t+\sigma)$ with $2 \leq i \leq 9$ as in the right-handed sector, and $X_{\mathrm{L}}^{I}(t+\sigma)$ with $1 \leq I \leq 16$. This decomposition gives rise eight combinations of ordinary physical coordinates, $X^{i}(\tau,\sigma)=X_{\mathrm{L}}^{i}(t+\sigma)+X_{\mathrm{R}}^{i}(t-\sigma)$. The additional left-moving coordinates $X_{\mathrm{L}}^{I}(t+\sigma)$ are responsible for the gauge symmetries G_{het} , cf. Equation (5).

光锥规范下，存在 8 个右行玻色弦坐标 $X_{\mathrm{R}}^{i}(t-\sigma)$ 和 8 个右行费米子 $\psi_{\mathrm{R}}^{i}(t-\sigma)$ ，其中 $2 \leq i \leq 9$.t 和 σ 表示世界面坐标。总共有 24 个左行坐标 $X_{\mathrm{L}}^{M}(t+\sigma)$ 。在对称紧化中，它们被分解为和右手征 sector 一样的 $X_{\mathrm{L}}^{i}(t+\sigma)$ (带 $2 \leq i \leq 9$) 以及带 $1 \leq I \leq 16$ 的 $X_{\mathrm{L}}^{I}(t+\sigma)$ 。该分解得到了 8 组普通物理坐标 $X^{i}(\tau,\sigma)=X_{\mathrm{L}}^{i}(t+\sigma)+X_{\mathrm{R}}^{i}(t-\sigma)$ 。额外的左行坐标 $X_{\mathrm{L}}^{I}(t+\sigma)$ 对应规范对称性 G_{het} ，参见式 (5)。

For the sake of keeping this chapter short, we specialize on symmetric compact-ifications, cf. Fig. 1. Interestingly, the so-called Free Fermionic Formulation (FFF) and $\mathbb{Z}_2 \times \mathbb{Z}_2$ geometric orbifolds are related by a dictionary [4,35]. There are possibilities to go more general, and consider, e.g., asymmetric orbifolds [55,73] or Gepner models [41,42], which is beyond the scope of this chapter.

为了精简本章篇幅，我们专门讨论对称紧化，参见图 1。有意思的是，所谓的自由费米子表述 (FFF) 和 $\mathbb{Z}_2 \times \mathbb{Z}_2$ 几何轨道面之间存在对应字典 [4,35]。当然也可以讨论更一般的情况，比如考虑不对称轨道面 [55,73] 或格普纳模型 [41,42]，这些内容超出了本章的范围。

Heterotic Strings on Orbifolds

轨道面上的杂化弦

We will start our discussion with heterotic orbifolds [28, 29], which allow one to explicitly ”see” the strings. For simplicity, we focus on symmetric toroidal orbifolds, which emerge by dividing tori by some of their symmetries. The tori are given by $\mathbb{R}^n/\Lambda$ or $\mathbb{C}^{n/2}/\Lambda$, where Λ denotes a lattice, or, more precisely, the group of lattice translations. We will be interested in $n=6$. Therefore, $\mathbb{O}=\mathbb{R}^n/\mathbb{S}$ with $\mathbb{S}$ denoting the so-called space group, which is comprised of lattice translations and additional operations such as rotations and so-called roto-translations and forms a discrete subgroup of the n -dimensional Euclidean group. Crucially, these operations are also embedded in the gauge sector, which breaks G_{het} down to a subgroup. Moreover, they also break SUSY, which facilitates the construction of chiral 4D models with $\mathcal{N}=1$ or no SUSY.

我们将从杂化轨道面 [28, 29] 开始讨论，它能让我们直观地“看到”弦。为简化分析，我们聚焦对称环面轨道面，这类轨道面是通过将环面按其部分对称性做商得到的。环面由 $\mathbb{R}^n/\Lambda$ 或 $\mathbb{C}^{n/2}/\Lambda$ 给出，其中 Λ 表示格，更准确地说，是格平移群。我们的研究对象是 $n = 6$ 。因此， $\mathbb{O} = \mathbb{R}^n/\mathbb{S}$ ，其中 $\mathbb{S}$ 是所谓的空间群，由格平移、旋转、旋转平移这类额外操作构成，是 n 维欧几里得群的一个离散子群。关键在于，这些操作也被嵌入规范场部分，将 G_{het} 破缺到一个子群。此外，它们还会破缺超对称 (SUSY)，这便于构建带有 $\mathcal{N} = 1$ 超对称或不带有超对称的四维手征模型。

In more detail, in the geometric formulation elements of space group $\mathbb{S}$ are conveniently denoted by $g = (\vartheta^r, m_\alpha e_\alpha)$, where $r, m_\alpha \in \mathbb{N}_0$. The e_α $(1 \le \alpha \le 6)$ are the basis vectors of the underlying torus. The set of $\vartheta \in \mathrm{O}(n)$ form a finite group, called the point group $\mathbb{P}$, which determines the holonomy group, and thus the amount of SUSY surviving the compactification. In fact, in order to classify the physically inequivalent orbifolds, one only needs to find the different affine classes [40], but we refrain from spelling this discussion out in detail. If $\mathbb{P}$ is Abelian, it is either $\mathbb{Z}_N$ or $\mathbb{Z}_N \times \mathbb{Z}_M$. If in addition $\mathcal{N} \ge 1$ SUSY is preserved, then a given $\mathbb{Z}_N$ transformation can be encoded in a so-called three-component twist vector v , which describes the rotations of three complex coordinates. In general, g acts on string coordinates X as

更具体地说，在几何表述中，空间群 $\mathbb{S}$ 的元素可方便地记为 $g = (\vartheta^r, m_\alpha e_\alpha)$ ，其中 $r, m_\alpha \in \mathbb{N}_0$ 。 e_α $(1 \le \alpha \le 6)$ 是底环面的基矢。所有 $\vartheta \in \mathrm{O}(n)$ 构成一个有限群，称为点群 $\mathbb{P}$ ，它决定了和乐群，进而决定了紧致化后剩余的超对称数量。实际上，要对物理上不等价的轨道面分类，我们只需要找出不同的仿射等价类 [40]，此处我们不展开详细讨论。若 $\mathbb{P}$ 是阿贝尔群，则它要么是 $\mathbb{Z}_N$ 要么是 $\mathbb{Z}_N \times \mathbb{Z}_M$ 。如果进一步保留 $\mathcal{N} \ge 1$ 超对称，那么给定的 $\mathbb{Z}_N$ 变换可以用所谓的三分量扭向量 v 编码，它描述了三个复坐标的旋转。一般情况下，g 对弦坐标 X 的作用为

$$X \xmapsto{(\vartheta^r, m_\alpha e_\alpha)} \vartheta^r X + m_\alpha e_\alpha. \tag{6}$$

The space group is to be embedded in the gauge degrees of freedom. Loosely speaking, the point group elements get mapped to so-called shift vectors V . This embedding has to preserve the order, i.e., if $\vartheta \in P$ satisfies $\vartheta^N = \mathbb{1}$ then $NV \in \Lambda_{\mathfrak{g}_{\text{het}}}$, where $\Lambda_{\mathfrak{g}_{\text{het}}}$ denotes the root lattice of G_{het} , cf. Equation (5). In $\mathbb{Z}_N$ orbifolds, if, say, the shifts of two models differ by lattice vectors $\lambda \in \Lambda_{\mathfrak{g}_{\text{het}}}$, the resulting models are identical (cf. [44]). This is no longer true in $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds, where gauge embeddings differing by lattice vectors may be inequivalent [79] and be related via what is known as discrete torsion [90]. Since $\Lambda_{\mathfrak{g}_{\text{het}}}$ is even and self-dual, one can find an Euclidean basis in which the lattice vectors are given by

空间群需要被嵌入规范自由度中。粗略来说，点群元素被映射为所谓的平移向量 V 。这种嵌入必须保留阶数，即若 $\vartheta \in P$ 满足 $\vartheta^N = \mathbb{1}$ ，则 $NV \in \Lambda_{\mathfrak{g}_{\text{het}}}$ ，其中 $\Lambda_{\mathfrak{g}_{\text{het}}}$ 是 G_{het} 的根格，参见式 (5)。在 $\mathbb{Z}_N$ 轨道面中，如果两个模型的平移相差格向量 $\lambda \in \Lambda_{\mathfrak{g}_{\text{het}}}$ ，那么得到的模型是等价的 (参见 [44])。这一点在 $\mathbb{Z}_N \times \mathbb{Z}_M$ 轨道面中不再成立: 相差格向量的规范嵌入可以是不等价的 [79]，它们可以通过所谓的离散挠率关联起来 [90]。由于 $\Lambda_{\mathfrak{g}_{\text{het}}}$ 是偶自对偶格，我们可以找到一个欧几里得基，使得格向量可写为

$$p = (n_1, \ldots, n_d) \quad \text{or} \quad p = (n_1 + 1/2, \ldots, n_d + 1/2)\,, \tag{7}$$

where $n_i \in \mathbb{Z}, \sum_i^d n_i \in 2\mathbb{Z}$ and $d \in \{8, 16\}$ denotes the dimensions of the Lie algebras E_8 or $\mathfrak{so}(32)$, respectively. The gauge embedding of each translation e_α is a so-called discrete Wilson line W_α . The Wilson lines are constrained by geometry. In more detail, since lattice vectors get mapped onto each other by the rotations, the analogous relations have to hold for the Wilson lines,

其中 $n_i \in \mathbb{Z}, \sum_i^d n_i \in 2\mathbb{Z}$ 和 $d \in \{8, 16\}$ 分别表示李代数 E_8 和 $\mathfrak{so}(32)$ 的维数。每个平移 e_α 的规范嵌入是所谓的离散威尔逊线 W_α。威尔逊线受几何约束。更具体地说，由于格矢会被旋转相互映射，威尔逊线也必须满足类似关系，

$$g e_\alpha = \sum_{\beta=1}^{6} a_\alpha{}^\beta e_\beta \;\Rightarrow\; W_\alpha = \sum_{\beta=1}^{6} a_\alpha{}^\beta W_\beta + \lambda \;\text{ with }\; \lambda \in \Lambda_{\mathfrak{g}_{\text{het}}}. \tag{8}$$

For instance, in a $\mathbb{Z}_3$ orbifold plane one has $\vartheta e_1 = e_2$ and $\vartheta e_2 = -e_1 - e_2$. Therefore, $W_1 \equiv W_2$ and $W_2 \equiv -W_1 - W_2$, where " $\equiv$ " means "equal up to $\lambda \in \Lambda_{\mathfrak{g}_{\text{het}}}$." Thus $3W_1 \in \Lambda_{\mathfrak{g}_{\text{het}}}$. This generalizes to other geometries, i.e., for a given Wilson line W_α there is an integer M_α such that $M_\alpha W_\alpha \in \Lambda_{\mathfrak{g}_{\text{het}}}$, with no summation over α . As a consequence, the coefficients $a_\alpha{}^\beta$ in (8) are integer.

例如，在一个 $\mathbb{Z}_3$ 轨形平面中满足 $\vartheta e_1 = e_2$ 和 $\vartheta e_2 = -e_1 - e_2$。因此可得 $W_1 \equiv W_2$ 和 $W_2 \equiv -W_1 - W_2$ ，其中“$\equiv$”意为“相差 $\lambda \in \Lambda_{\mathfrak{g}_{\text{het}}}$ 相等”。由此得 $3W_1 \in \Lambda_{\mathfrak{g}_{\text{het}}}$。这可以推广到其他几何结构，即对给定的威尔逊线 W_α ，存在整数 M_α 满足 $M_\alpha W_\alpha \in \Lambda_{\mathfrak{g}_{\text{het}}}$ ，不对 α 求和。因此，式 (8) 中的系数 $a_\alpha{}^\beta$ 为整数。

In addition, the orbifold parameters and their gauge embeddings must satisfy a series of constraints in order to ensure world-sheet modular invariance, which guarantees the UV consistency of the model [29, 90] . For $\mathbb{Z}_N$ orbifolds, these conditions take the form [79]

此外，轨形参数及其规范嵌入必须满足一系列约束条件，以保证世界面模不变性，这是模型 [29, 90] 紫外一致性的保证。对于 $\mathbb{Z}_N$ 轨形，这些条件可写为 [79]

$$N\left(V^2 - v^2\right) = 0 \bmod 2,\; M_\alpha V \cdot W_\alpha = 0 \bmod 2,$$

$$M_\alpha W_\alpha^2 = 0 \bmod 2,\; \gcd\left(M_\alpha, M_\beta\right) W_\alpha \cdot W_\beta = 0 \bmod 2, \tag{9}$$

where no summation over α nor β is implied.

其中不对 α 和 β 求和。

Classification of Toroidal Orbifold Geometries

环面轨形几何的分类

While early classifications of viable toroidal orbifolds focused on special kinds of lattices [15], more recently a richer set of possibilities has been uncovered in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold [35] and generalized to other point groups [39,40]. Loosely speaking, the new ingredient of the additional possibilities are space groups which contain roto-translations $(\vartheta^r, m_\alpha e_\alpha) \in \mathbb{S}$ but $(\vartheta^r, 0) \notin \mathbb{S}$ (cf. [52]). As a consequence, the fundamental group of the orbifolds (and not just the underlying tori) can be nontrivial. Among other things, this allows for non-local, or Wilson line, breaking of the gauge symmetry, which is also being utilized in the context of smooth compactifications [10]. Another innovation is the consistent construction of non-Abelian orbifolds [39, 60]. In particular, there are 138 Abelian and 331 non-Abelian space groups [40] of toroidal symmetric orbifolds preserving $\mathcal{N} = 1$ SUSY in 4D. These geometries have been shown to host many models with gauge symmetry and chiral spectrum of the MSSM [74, 76, 81], yet their detailed phenomenological properties have not been worked out so far.

尽管早期对可行环面轨形的分类聚焦于特殊晶格类型 [15]，近来在 $\mathbb{Z}_2 \times \mathbb{Z}_2$ 轨形中发现了更多丰富的可能性，并被推广到其他点群 [39,40]。泛泛而言，这些额外可能性的新要素是包含旋转平移 $(\vartheta^r, m_\alpha e_\alpha) \in \mathbb{S}$ 但满足 $(\vartheta^r, 0) \notin \mathbb{S}$ 的空间群 (参见 [52])。因此，轨形的基本群 (不仅是底环面的基本群) 可以是非平凡的。除此之外，这允许规范对称性发生非定域破缺，即威尔逊线破缺，该方法也已应用于光滑紧化的研究中 [10]。另一项创新是构造出自洽的非阿贝尔轨形 [39, 60]。具体而言，四维中保持 $\mathcal{N} = 1$ 超对称的环面对称轨形共有 138 种阿贝尔空间群和 331 种非阿贝尔空间群 [40]。已经证明这些几何中存在大量具有最小超对称标准模型规范对称性和手征谱的模型 [74, 76, 81]，但到目前为止，它们的详细唯象性质尚未得到研究。

Anisotropic Compactifications

各向异性紧致化

Because of the gauge symmetries of the heterotic string, heterotic models comply well with the idea of grand unification. Breaking the GUT symmetry via compactification allows one to elegantly avoid the major shortcomings of 4D GUTs, most notably the doublet-triplet splitting challenge and its associated proton decay problems. However, there is a tension between the scale of gauge coupling unification in the MSSM, $M_{\mathrm{GUT}} \simeq$ few $\cdot 10^{16}$GeV , and typical compactification radii. This is because string theory also describes gravity, and the effective 4D Planck mass is sensitive to the volume of compact space. In some more detail, Newton's constant G_{N} is related to the fine structure "constant" at the GUT/compactification scale, α_{GUT} , and the string tension, α' , via [91]

由于杂弦具有规范对称性，杂弦模型很好地符合大统一的思想。通过紧致化破坏大统一理论对称性，可以巧妙规避四维大统一理论的主要缺陷，最突出的就是双态-三态分裂问题及其相关的质子衰变问题。但最小超对称标准模型中规范耦合统一的能标 $M_{\mathrm{GUT}} \simeq$ few $\cdot 10^{16}$GeV 与典型紧致化半径之间存在矛盾。这是因为弦理论也描述引力，四维有效普朗克质量对紧致空间的体积很敏感。更具体来说，牛顿常数 G_{N} 与大统一/紧致化能标下的精细结构“常数”α_{GUT} 以及弦张力 α' 通过下式关联 [91]

$$G_{\mathrm{N}} = \frac{\alpha_{\mathrm{GUT}}\alpha'}{4} \quad \text{implying that } G_{\mathrm{N}} \gtrsim \frac{\alpha_{\mathrm{GUT}}^{4/3}}{M_{\mathrm{GUT}}^2} \tag{10}$$

for a weakly coupled theory. This value of G_{N} is too large for typical values of M_{GUT} and α_{GUT} (cf. our earlier discussion around Equation (4)). There are various proposals to fix this issue (see, e.g., [30]). The perhaps most ingenious way to address this problem is M-theory [91]. However, the problem can also be ameliorated in anisotropic compactifications [91, Footnote 3]. A detailed analysis [52] suggests that this solution barely fails, but by the own admission of the authors the presented bound is too conservative. In fact, if one uses the appropriate volume of the orbifold for the analysis rather than the underlying torus, one finds that anisotropic compactifications can work, even though the parameter space of solutions is not too generous. This implies that there is an intermediate orbifold GUT symmetry (see, e.g., [82] for a review). However, this also means that the smaller radii are of the order of the string scale, and as stressed in [91], one must use conformal field theory (CFT) (rather than classical geometry) to analyze the model. This is one of the reasons why this chapter focuses on orbifold constructions.

这是弱耦合理论的情况。对于 M_{GUT} 和 α_{GUT} 的典型取值，该 G_{N} 值过大 (参见我们之前围绕方程 (4) 的讨论)。目前已有多种解决该问题的方案 (例如参见 [30])。解决这个问题最巧妙的方法或许是 M 理论 [91]。但该问题也可以在各向异性紧致化中得到缓解 [91, 脚注 3]。一项详细分析 [52] 表明该方案勉强不成立，但作者自己也承认文中给出的界过于保守。事实上，如果分析时采用轨道面而非其底环面的正确体积，就会发现即使解的参数空间并不充裕，各向异性紧致化仍然可行。这说明存在中间轨道面大统一对称性 (综述参见例如 [82])。但这也意味着更小的半径与弦尺度同阶，正如文献 [91] 中强调的，必须使用共形场论 (CFT) 而非经典几何来分析该模型。这也是本章聚焦轨道面构造的原因之一。

Spectrum

谱

Given a compactification of the heterotic string, one can determine its spectrum, i.e., the properties of the massless and massive excitations of strings, see (Fig. 2). One usually proceeds in two steps, by first determining the spectrum "after compactification" and then the spectrum of deformation of the model in which certain VEVs get switched on. In this section we focus on the former.

给定杂弦的一种紧化，我们可以确定其能谱，即弦的无质量和有质量激发的性质，参见 (图 2)。通常分为两步进行: 首先确定“紧化后”的能谱，随后确定开启某些真空期望值后模型的形变能谱。本节我们聚焦于前者。

Massless Gauge Fields

无质量规范场

In general, only a subset of the G_{het} gauge fields survive the orbifold projections. They can be determined by finding the roots $p \in \Lambda_{\mathfrak{g}_{\text{het}}}$, i.e., $p \cdot p = 2$, which satisfy the projection conditions

一般而言，仅 G_{het} 规范场的一个子集能留存过轨形投影。我们可以通过找出满足投影条件的根 $p \in \Lambda_{\mathfrak{g}_{\text{het}}}$ ，即 $p \cdot p = 2$ ，来确定这些规范场

$$p \cdot V = p \cdot W = 0 \,(\bmod\ 1) \tag{11}$$

with the p from Equation 7 for all shift V and Wilson line W vectors (cf. section "Heterotic Strings on Orbifolds").

其中 p 取自式 (7)，适用于所有平移 V 和威尔逊线 W 矢量 (参见“轨形上的杂化弦”一节)。

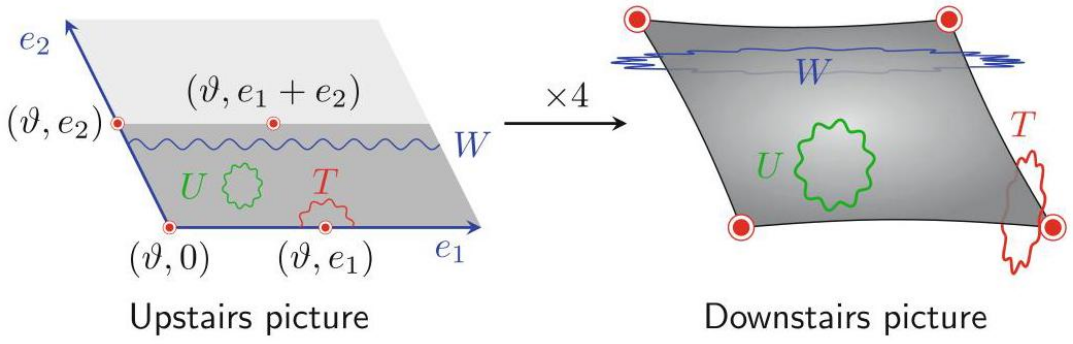

Fig. 2 Cartoon of an orbifold. Strings on orbifolds are in one-to-one correspondence with the conjugacy classes of the space group \$. Untwisted and twisted strings are associated with elements with trivial and nontrivial rotations, respectively. In the depicted example of a $\mathbb{T}^2/\mathbb{Z}_2$ orbifold, the fundamental domain is half of the fundamental domain of the torus. The edges can be identified (or ”glued together”) to obtain a pillow in the downstairs picture, where the twisted strings T are localized at the corners. Untwisted strings U are free to propagate in the bulk, and winding strings W wind around the torus

图 2 轨形示意图。轨形上的弦与空间群 \$ 的共轭类一一对应。未扭弦和扭弦分别对应平凡旋转元和非平凡旋转元。图中所示为一个 $\mathbb{T}^2/\mathbb{Z}_2$ 轨形的例子，其基本域是环面基本域的一半。将边对齐 (即“粘合在一起”) 就得到下图中的枕形，扭弦 T 局域在角上。未扭弦 U 可在体中自由传播，缠绕弦 W 环绕环面缠绕

Chiral Zero Modes

手征零模

The zero modes are solutions of the mass equation to vanishing mass. In all known examples they are chiral w.r.t. some, possibly discrete, symmetry. The computation of the massless spectrum, i.e., gauge and chiral zero modes, is straightforward though tedious if done by hand and can conveniently be performed with dedicated tools such as the Orbifolder [70].

零模是质量方程中满足质量为零的解。在所有已知例子中，它们相对于某种 (可能是离散的) 对称性是手征的。对无质量谱即规范零模与手征零模的计算十分直接，但手动操作十分繁琐，也可以借助专用工具 (比如 Orbifolder [70]) 方便地完成计算。

Explicit string models exhibit states beyond the SM or MSSM at some level. The additional states include the moduli as well as the winding and Kaluza-Klein (KK) modes, which we will review in sections ”Moduli” and ”Winding and KK Modes,” respectively. In addition, there are often vector-like states w.r.t. the SM gauge group which are neither KK nor winding modes. Whether or not these vector-like states are massless often depends on the point of moduli space under consideration. For instance, vector-like states may attain masses when giving VEVs to blow-up modes that smoothen out orbifold singularities and break symmetries w.r.t. which these states are chiral. However, it would be arguably wrong to refer to the smoothened out version as ”cleaner” since it is really the same model. In fact, often important properties of a given construction are much more directly accessible by studying the symmetry-enhanced point in field space, which is given by the orbifold point in this example, even though the vacuum is away from this point.

> 显式弦模型在某一能标下会出现超出标准模型 (SM) 或最小超对称标准模型 (MSSM) 的态。额外态包括模，以及缠绕模和卡鲁扎-克莱因 (KK) 模，我们将分别在“模”和“缠绕模与 KK 模”章节中对它们进行评述。此外，除 KK 模和缠绕模外，经常还存在相对于 SM 规范群的矢量类态。这些矢量类态是否无质量，通常取决于所考虑的模空间点。例如，当给爆胀模式赋予真空期望值 (VEV)，从而平滑轨道面奇点并破缺这些态所属的手征对称性时，矢量类态就会获得质量。但将平滑后的版本称为“更干净”的模型其实并不准确，因为它本质上仍是同一个模型。事实上，即便真空不在对称性增强点上，研究给定构造的重要性质时，研究场空间中的该点 (本例中即轨道面点) 往往能直接得多。

Moduli

模

Virtually every supersymmetric string compactification contains fields which are classically flat directions, and this is in particular true for models that come close to the real world. Some of these so-called moduli do not have any charge under G_{het} , and comprise the Kähler moduli $\mathcal{T}_i$, the complex structure moduli $\mathcal{U}_i$ and the dilaton $\mathcal{S}$. Yet also some of the charged fields can attain VEVs because they are along D -flat directions. The VEVs of these fields determine, among other things, geometric properties of compact space. Classical flat directions can attain a nontrivial potential at the quantum level, in particular through nonperturbative effects. It is generally challenging to compute these potentials in full detail and thus determine the VEVs at the minima, see Chap. 57, ”Moduli Stabilization in String Theory” for more details of the analogous discussion in other versions of string theory.

> 几乎所有超对称弦紧致化都包含经典层面的平坦方向场，对于近似真实世界的模型而言尤其如此。这些所谓的模中，有一部分在 G_{het} 下不带任何荷，包括凯勒模 $\mathcal{T}_i$ 、复结构模 $\mathcal{U}_i$ 以及胀子 $\mathcal{S}$ 。但即便部分带荷场也可以获得真空期望值，因为它们处于 D 平坦方向上。除其他因素外，这些场的真空期望值决定了紧致空间的几何性质。经典平坦方向在量子层面会产生非平凡势，尤其是通过非微扰效应产生。完整计算这些势并由此确定势极小点处的真空期望值通常十分困难，关于其他弦理论版本中的类似讨论，详见第 57 章“弦理论中的模稳定”。

Winding and KK Modes

缠绕模与 KK 模

The existence of winding and KK modes is one of the most important features of string compactifications and required to make the theory UV complete. The properties of these modes are particularly accessible in torus-based compactifications such as toroidal orbifolds.

缠绕模与 KK 模的存在是弦紧致化最重要的特征之一，也是让该理论在紫外区域完备的必要条件。在基于环面的紧致化 (例如环面轨形) 中，这些模的性质格外容易研究。

Symmetries of the Effective Action

有效作用量的对称性

Heterotic compactifications lead to effective 4D theories exhibiting various symmetries [32], which largely determine the phenomenological properties of the respective models. These symmetries include

杂弦紧致化会得到具有多种对称性的四维有效理论 [32]，这些对称性在很大程度上决定了相应模型的唯象性质。这些对称性包括

1. Either $\mathcal{N} = 1$ or no supersymmetry

1. $\mathcal{N} = 1$ 超对称，或无超对称

2. Continuous gauge symmetries, which mainly originate from the 10DG_{het} symmetries, i.e., the root lattice of the 16 left-moving coordinates

2. 连续规范对称性，主要起源于 10DG_{het} 对称性，即 16 个左动坐标的根格

3. R symmetries (in SUSY compactifications)

3. R 对称性 (存在于超对称紧致化中)

4. Flavor symmetries

4. 味对称性

5. Modular symmetries

5. 模对称性

6. Outer automorphisms which may be $\mathcal{CP}$ or $\mathcal{CP}$ -like transformations (It is important to distinguish between proper $\mathcal{CP}$ transformations, which map all particles to their own antiparticles, and $\mathcal{CP}$ -like transformations, which only send some of the particles to their antiparticles [23].)

6. 外自同构可以是 $\mathcal{CP}$ 变换或类 $\mathcal{CP}$ 变换 (区分将所有粒子映射到其自身反粒子的正宗 $\mathcal{CP}$ 变换，与仅将部分粒子映射到其反粒子的类 $\mathcal{CP}$ 变换十分重要 [23]。)

More recently, it has been pointed out that the symmetries 3-6 may be regarded as outer automorphisms of the Narain lattice [17, 18] in the Narain formulation of toroidal compactifications of the heterotic strings [69] (see also [49]). The gauge symmetries can go beyond G_{het} if the compact space has special properties, such as some radii equalling certain critical values (cf. e.g., [49]).

近来已有研究指出，在杂化弦环面紧化的 Narain 表述中，3-6 号对称性可被视为 Narain 晶格的外自同构 [17, 18] [69](另见 [49])。若紧化空间具备特殊性质，例如部分半径等于特定临界值 (参见例如 [49])，则规范对称性可以超出 G_{het} 的范围。

SUSY

超对称 (SUSY)

$\mathcal{N} = 1$ SUSY (cf. section "Beyond the Standard Model (BSM) Scenarios") has long been a standard ingredient of string models. Whether or not $\mathcal{N} = 1$ SUSY is preserved by the compactification depends on the holonomy group of the compact space [24]. In the case of a smooth compactification, the requirement that the compactification preserves $\mathcal{N} = 1$ SUSY dictates that the manifold has to be of the Calabi-Yau (CY) type, and in orbifolds it requires the twist to fit into SU(3), the holonomy group of CY manifolds.

$\mathcal{N} = 1$ 超对称 (参见“超出标准模型 (BSM) 场景”章节) 长期以来一直是弦模型的标准组成部分。紧致化是否保留 $\mathcal{N} = 1$ 超对称，取决于紧致空间的和乐群 [24]。对于光滑紧致化，要求紧致化保留 $\mathcal{N} = 1$ 超对称就规定了流形必须是卡拉比-丘 (CY) 型，而在轨形中则要求扭转适配 SU(3)，即卡拉比-丘流形的和乐群。

Continuous Gauge Symmetries

连续规范对称性

After compactification the residual continuous gauge symmetry, G_{gauge}, of a realistic model has to contain the gauge symmetry of the SM (1). The gauge symmetry follows already from our discussion in section "Massless Gauge Fields." Apart from the obvious option that $G_{\text{gauge}} = G_{\text{SM}} \times G_{\text{beyond}}$ promising models may also replace G_{SM} by the Pati-Salam [80] (PS) group $G_{\text{PS}} = \text{SU}(4)_{\text{C}} \times \text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}}$, the so-called left-right symmetry [87] $G_{\text{LR}} = \text{SU}(3)_{\text{C}} \times \text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}} \times \text{U}(1)_{B-L}$, or the flipped SU(5) symmetry [8]. Other grand unified symmetries are in principle possible but may be challenged by doublet-triplet splitting problems and the lack of appropriate Higgs fields that break the larger symmetry to G_{SM}.

紧致化后, 现实模型的剩余连续规范对称性 G_{gauge} 必须包含标准模型 (1) 的规范对称性。我们在“无质量规范场”一节的讨论已经推导出了规范对称性。除了显而易见的方案 $G_{\text{gauge}} = G_{\text{SM}} \times G_{\text{beyond}}$ 之外，有前景的模型也可以将 G_{SM} 替换为帕蒂-萨拉姆 [80](PS) 群 $G_{\text{PS}} = \text{SU}(4)_{\text{C}} \times \text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}}$，即所谓的左右对称性 [87] $G_{\text{LR}} = \text{SU}(3)_{\text{C}} \times \text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}} \times \text{U}(1)_{B-L}$，或者翻转 SU(5) 对称性 [8]。原则上其他大统一对称性也是可行的，但会面临双态-三态分裂问题，且缺乏合适的希格斯场将更大的对称性破缺为 G_{SM}。

Very often in geometric orbifolds with $\mathcal{N} = 1$ SUSY one U(1) factor appears anomalous, with the anomaly being cancelled by the Green-Schwarz [50] (GS) mechanism. As a consequence, the D-term potential contains an Fayet-Iliopoulos [36] (FI) term, $\mathcal{V}_D \supset g^2 D^2_{\text{anom}}$, where

在具有 $\mathcal{N}=1$ 超对称的几何轨道紧化中，经常会出现一个 U(1) 因子是反常的，该反常可以通过格林-施瓦茨 [50](GS) 机制抵消。因此，D 项势会包含费耶-伊利亚普洛斯 [36](FI) 项 $\mathcal{V}_D \supset g^2 D^2_{\text{anom}}$ ，其中

$$D_{\text{anom}} = \sum_i q^i_{\text{anom}} |\varphi_i|^2 + \xi = 0 \text{ with } \xi = \frac{g^2 \operatorname{Tr} t_{\text{anom}}}{192\pi^2} M_{\mathrm{P}}^2. \tag{12}$$

g denotes the gauge coupling, and t_{anom} the generator of $\mathrm{U}(1)_{\text{anom}}$. The requirement of a vanishing of the D -term potential induces VEVs of $\mathrm{U}(1)_{\text{anom}}$ charged fields φ_i that breaks $\mathrm{U}(1)_{\text{anom}}$ and in the overwhelming majority of cases further symmetries [37]. Clearly, in realistic models the fields φ_i acquiring large VEVs must be SM singlets. Configurations with vanishing D -terms can be identified by constructing holomorphic monomials which are invariant under all gauge symmetries but carry nontrivial charge under $\mathrm{U}(1)_{\text{anom}}$ [11, 21, 22] . A complete basis of such monomials can be obtained via the Hilbert basis [62]. In the vast majority of explicit models, $\operatorname{Tr} t_{\text{anom}} \sim O(100)$, so that $\sqrt{\xi}/M_{\mathrm{P}}$ is of the order of the Cabibbo angle, and, therefore, the VEVs induced by (12) may conceivably play a role in explaining flavor hierarchies [20].

g 表示规范耦合，t_{anom} 表示 $\mathrm{U}(1)_{\text{anom}}$ 的生成元。要求 D 项势为零会诱导 $\mathrm{U}(1)_{\text{anom}}$ 荷场 φ_i 获得真空期望值 (VEV), 从而破缺 $\mathrm{U}(1)_{\text{anom}}$ ，在绝大多数情况下还会进一步破缺其他对称性 [37]。显然, 在现实模型中, 获得大真空期望值的场 φ_i 必须是标准模型单态。我们可以通过构造在所有规范对称性下不变、但在 $\mathrm{U}(1)_{\text{anom}}$ [11, 21, 22] 下携带非平凡荷的全纯单项式，来确定 D 项为零的构型。这类单项式的完全基可以通过希尔伯特基得到 [62]。在绝大多数已构建的显式模型中，$\operatorname{Tr} t_{\text{anom}} \sim O(100)$ ，因此 $\sqrt{\xi}/M_{\mathrm{P}}$ 是卡比博角量级，因此 (12) 诱导的真空期望值完全有可能在解释味层级中发挥作用 [20]。

The extra symmetry G_{beyond} is usually partly broken by the VEVs that cancel the FI term. The residual continuous part can conceivably provide us with a hidden sector leading to dynamical SUSY breakdown [31, 56]. There are also usually discrete symmetries, which can be determined systematically with the Smith normal form [78].

额外对称性 G_{beyond} 通常会被抵消 FI 项的真空期望值部分破缺。剩余的连续部分可以为我们提供一个隐藏域，从而引发动力学超对称破缺 [31, 56]。通常还会存在离散对称性，这类对称性可以通过史密斯标准型系统确定 [78]。

Discrete Symmetries

离散对称性

Symmetries From a Narain Compactification

纳拉因紧致化的对称性

The Narain formulation provides an alternative to the usual toroidal compactification of the heterotic string discussed in section ”Compactifying the Heterotic String.” Let us consider first scenarios in which the six extra dimensions are compactified in a $\mathbb{T}^6$. In the Narain formulation, the 6 right- and 6+16 left-moving compact (bosonic) coordinates are considered independently, so that, taking into account also the gauge degrees of freedom, the $\mathbb{T}^6$ compactification is specified in terms of an auxiliary $(2 \cdot 6 + 16)$ D torus according to

纳拉因表述是“杂化弦紧致化”一节中介绍的常规环面杂化弦紧致化的替代方案。我们首先考虑六个额外维度按照 $\mathbb{T}^6$ 紧致化的情形。在纳拉因表述中，6 个右行和 6+16 个左行紧致 (玻色) 坐标被视为独立量，因此同时考虑规范自由度后，$\mathbb{T}^6$ 紧致化可通过辅助 $(2 \cdot 6 + 16)$ D 环面按下文定义

$$Y \sim Y + E\widehat{N}, \quad \text{where} \quad Y = \left(X_R^i, X_L^i, X_L^I\right)^{\mathrm{T}}, \; \widehat{N} = (w, k, q)^{\mathrm{T}} \in \mathbb{Z}^{2.6+16}.$$

(13)

Here, E is the Narain vielbein which spans the 28D even, integer, and self-dual Narain lattice Γ of signature $(6, 6+16)$. Further, the integer vector $\widehat{N}$ includes the winding numbers w^i , the KK numbers k^i , and the gauge momenta q^I discussed in section ”Continuous Gauge Symmetries.” The properties of Γ are encoded in the condition

此处，E 是张成符号差为 $(6, 6+16)$ 的 28D 偶、整、自对偶 Narain 格 Γ 的 Narain 标架。此外，整数向量 $\widehat{N}$ 包含了章节“连续规范对称性”中讨论的绕数 w^i 、KK 数 k^i 和规范动量 q^I 。Γ 的性质被编码在如下条件中

$$E^{\mathrm{T}}\eta E = \begin{pmatrix} 0 & \mathbb{1}_D & 0 \\ \mathbb{1}_D & 0 & 0 \\ 0 & 0 & g \end{pmatrix} =: \hat{\eta}, \tag{14}$$

where η is the flat metric with signature $(6, 6+16)$, g denotes the 16D Cartan matrix of the heterotic string, and we have defined the Narain metric $\hat{\eta}$.

其中符号为 $(6, 6+16)$, g 的 η 是平坦度量，表示杂化弦的 16 维嘉当矩阵，我们已经据此定义了纳拉因度量 $\hat{\eta}$ 。

It turns out that the group of outer automorphisms of Γ , defined by

可以证明，Γ 的外自同构群可由下式定义

$$\mathrm{O}_{\hat{\eta}}(6, 6+16, \mathbb{Z}) = \left\{\widehat{\sum} \mid \widehat{\sum}^{\mathrm{T}} \hat{\eta} \widehat{\sum}\right\} \quad \text{with} \quad \widehat{\sum} \in \mathrm{GL}(2 \cdot 6 + 16, \mathbb{Z}), \tag{15}$$

describes all the discrete symmetries of the toroidal compactification. Hence, naturally $\mathrm{O}_{\hat{\eta}}(6, 6+16, \mathbb{Z})$ contains the modular transformations of the compactification, including mirror symmetries and $\mathcal{CP}$ -like transformations of the moduli.

它描述了环面紧致化的所有离散对称性。因此自然地，$\mathrm{O}_{\hat{\eta}}(6, 6+16, \mathbb{Z})$ 包含紧致化的模变换，包括镜对称性和模空间的 $\mathcal{CP}$ 类变换。

From this compactification, it is easy to arrive at the symmetries of a toroidal orbifold. In this formalism, an orbifold is obtained by modding out a subgroup of Γ . Let us consider, for simplicity, the case of an Abelian orbifold without roto-translations. Treating left- and right-moving coordinates as independent, as before, the orbifold identification is given by

通过这种紧致化，我们可以很容易地得到环面 orbifold(轨形) 的对称性。在此形式体系中，orbifold 通过商去 Γ 的一个子群得到。为简化讨论，我们考虑没有旋转平移的阿贝尔 orbifold 情形。和之前一样将左行与右行坐标视为独立量，orbifold 的认同条件为

$$Y \sim \Theta^r Y + E\hat{N} \tag{16}$$

with the Narain twist

其中用到纳拉因扭转

$$\Theta = \mathrm{diag}\left(\vartheta_{\mathrm{R}}, \vartheta_{\mathrm{L}}, \vartheta_g\right), \quad \text{where } \vartheta_{\mathrm{R}}, \vartheta_{\mathrm{L}} \in \mathrm{O}(6) \text{ and } \vartheta_g \in \mathrm{O}(16). \tag{17}$$

We can further impose the orbifold to be of order N by demanding $\Theta^N = \mathbb{1}$. The Narain twist must leave the chosen Narain lattice invariant, i.e., $\Theta\Gamma = \Gamma$, which ensures that the moduli remain invariant under the orbifold action. Hence, some of the moduli of the original toroidal compactification are hereby fixed. Note that the possibility $\vartheta_{\mathrm{L}} \neq \vartheta_{\mathrm{R}}$ defines an asymmetric orbifold. Limiting ourselves to $\vartheta_{\mathrm{L}} = \vartheta_{\mathrm{R}} = \vartheta$, we recover the geometric picture of the symmetric orbifolds introduced in section ”Compactifying the Heterotic String.”

我们可以进一步通过要求 $\Theta^N = \mathbb{1}$ ，限定 orbifold 的阶为 N 。纳拉因扭转必须保持所选的纳拉因格不变，即满足 $\Theta\Gamma = \Gamma$ ，这保证了模在 orbifold 作用下保持不变。因此原环面紧致化的部分模会被固定。注意 $\vartheta_{\mathrm{L}} \neq \vartheta_{\mathrm{R}}$ 的可能性对应不对称 orbifold。当限定为 $\vartheta_{\mathrm{L}} = \vartheta_{\mathrm{R}} = \vartheta$ 时，我们就得到了“杂化弦紧致化”一节中介绍的对称 orbifold 的几何图像。

The discrete symmetries of the orbifold include then the subgroup of rotational outer automorphisms of the toroidal compactification, $\widehat{\sum} \in \mathrm{O}_{\hat{\eta}}(6, 6+16, \mathbb{Z})$, that are left unbroken by the orbifold, i.e., which satisfy

因此 orbifold 的离散对称性包含环面紧致化中未被 orbifold 破缺的旋转外自同构子群 $\widehat{\sum} \in \mathrm{O}_{\hat{\eta}}(6, 6+16, \mathbb{Z})$ ，即满足下式的自同构

$$\widehat{\sum}^{-1} \hat{\Theta}^k \widehat{\sum} = \hat{\Theta}^{k'}, \quad \text{where } k, k' = 1, \ldots, N, \tag{18}$$

and $\hat{\Theta} = E^{-1}\Theta E$ is the Narain twist in the Narain lattice basis. In addition, now there are translational outer automorphisms of the orbifold (The Narain twist combines with the translations of the Narain lattice to build the Narain space group $\mathbb{S}_{\mathrm{Narain}}$. Formally, it is the outer automorphisms of S_{Narain} that we refer here as the automorphisms of the orbifold.) given by

且 $\hat{\Theta} = E^{-1}\Theta E$ 是纳拉因格基下的纳拉因扭转。此外，orbifold 还存在平移外自同构 (纳拉因扭转与纳拉因格的平移结合构成纳拉因空间群 $\mathbb{S}_{\mathrm{Narain}}$ 。形式上，我们此处所说的 orbifold 自同构就是 S_{Narain} 的外自同构。) 由下式给出

$$Y \sim Y + E\hat{T}, \quad \text{with } \hat{T} \notin \mathbb{Z}^{2\cdot 6+16}. \tag{19}$$

In order to be compatible with the orbifold, the translations must fulfill

为了与 orbifold 相容，平移必须满足

$$\left(\mathbb{1}_{2\cdot 6+16} - \widehat{\Theta}^k\right)\widehat{T} \in \Gamma,\ 1 \le k \le N. \tag{20}$$

Note that these translations build a normal subgroup of the full group of outer automorphism of the orbifold.

注意这些平移构成了 orbifold 整个外自同构群的正规子群。

These discrete residual transformations give rise to R , flavor, modular, and outer-automorphism symmetries, which we will discuss separately in what follows.

这些剩余离散变换会引出 R 、味对称性、模对称性和外自同构对称性，我们将在后续分别讨论。

R Symmetries

R 对称性

Supersymmetric orbifold compactifications usually do not break the Lorentz symmetry of the compact six dimensions completely but leave discrete remnants which act as R symmetries in the effective description. Since the superpotential has a nontrivial modular weight, modular transformations, which we will discuss in more detail below in section "Modular Symmetries," are generically R symmetries. As we shall see in an explicit example in section "Geometric Orbifold with $\mathcal{N} = 1$ SUSY," certain R symmetries can be instrumental in resolving some of the phenomenological issues.

超对称轨道紧致化通常不会完全破缺六个紧致维度的洛伦兹对称性，而是会留下离散残余，这些残余在有效描述中作为 R 对称性发挥作用。由于超势具有非平凡模权重，我们将在下文“模对称性”小节中详细讨论的模变换，一般来说都是 R 对称性。正如我们将在“具有 $\mathcal{N} = 1$ 超对称的几何轨道”小节的具体例子中看到的那样，特定的 R 对称性可以帮助解决部分唯象学问题。

Flavor Symmetries

味对称性

The repetition of families in the SM begs for an explanation. Flavor symmetries may address this question. String compactifications can give rise to non-Abelian discrete symmetries in which the three generations of the SM transform as a 3-plet, or two generations as a 2-plet. Such symmetries may arguably play a role in understanding the flavor structure of the SM (see, e.g., [53] for references).

粒子物理标准模型中费米子代的重复存在亟待解释，味对称性或许可以解答这个问题。弦紧致化可以产生非阿贝尔离散对称性，标准模型的三个代可以变换为一个三重态，两个代变换为一个二重态。这类对称性可以说对理解标准模型的味结构有重要作用 (相关引用参见例如文献 [53])。

In the geometric approach, flavor symmetries can be obtained from the replication of matter states at different yet equivalent orbifold singularities in the compact dimensions [79]. The emerging permutations combine with additional symmetries from the string selection rules to non-Abelian discrete symmetries. These rules act on matter fields as Abelian symmetries of the effective theory, which can be understood as an Abelianization of the space group of the orbifold [85]. It has been verified in explicit examples that the above-mentioned non-Abelian symmetries emerge from continuous gauge symmetries [14] are hence gauged, as one would expect.

在几何方法中，味对称性可以来源于紧致维度中不同但等价的轨道奇点处物质态的重复出现 [79]。由此产生的置换与弦选择规则带来的额外对称性结合，形成非阿贝尔离散对称性。这些选择规则作为有效理论的阿贝尔对称性作用在物质场上，可以理解为轨道空间群的阿贝尔化 [85]。已有具体例子验证，上述非阿贝尔对称性来源于连续规范对称性 [14]，因此正如预期是被规范的。

In the Narain formalism, these symmetries are identified with the subgroup of translational outer automorphisms of the orbifold.

在 Narain 形式中，这些对称性对应于轨道平移外自同构的子群。

Modular Symmetries

模对称性

Modular symmetries are ubiquitous in string compactifications. They are symmetries of certain loop diagrams and the partition function. Moreover, toroidal orbifold compactifications exhibit modular symmetries. It is important to distinguish between the two.

模对称性在弦紧化中普遍存在。它们是特定圈图和配分函数的对称性。此外，环面轨形紧化也展现出模对称性。区分这两种模对称性十分重要。

World-sheet modular invariance has far-reaching implications for the UV consistency of the theory, the comprehensive discussion of which is beyond the scope of this chapter. In particular, modular invariance conditions constrain the choices of the geometrical data of the models [29, 90]. Among other things, they ensure that the models are free of anomalies.

世界面模不变性对理论的紫外一致性影响深远，相关全面讨论超出本章范围。模不变性条件尤其会约束模型几何数据的选择 [29, 90]，还能保证模型没有反常。

Target-space modular invariance provides us with important constraints on the Kähler potential and couplings of the theory [64]. These modular symmetries contain crucial information on the couplings of the theory [38] and even provide us with an alternative to the CFT computation [26].

靶空间模不变性为理论的凯勒势和耦合给出了重要约束 [64]。这些模对称性包含理论耦合的关键信息 [38]，甚至可以为共形场论计算提供替代方案 [26]。

In the geometric approach of toroidal orbifold compactifications, the properties of target-space modular symmetries have been explored. Among other features, these symmetries are free of anomalies thanks to the GS mechanism. Further, the transformation of matter fields under these symmetries have been determined. Denoting by (Here, we adopt the convention $T = \frac{1}{\alpha'}\left(B + \mathrm{i}\sqrt{\det G}\right)$, where B is the nontrivial component of the antisymmetric B -field and G the metric of the 2D orbifold sector, respectively.) T the Kähler modulus of a $\mathbb{T}^2/\mathbb{Z}_N$ orbifold sector, by γ any transformation from the corresponding $\mathrm{SL}(2,\mathbb{Z})$ modular group, the p th multiplet Φ_p of twisted matter fields of the orbifold transform according to [65, 66]

在环面轨形紧化的几何方法中，人们已经探究了靶空间模对称性的性质。这些对称性的特点之一是，借助格林-施瓦茨机制消除了反常。此外，物质场在这些对称性下的变换也已经确定。记 $T = \frac{1}{\alpha'}\left(B + \mathrm{i}\sqrt{\det G}\right)$ (此处我们采用该约定，其中 B 是反对称 B 场的非平凡分量，G 分别是二维轨形 sector 的度规) 中 T 为 $\mathbb{T}^2/\mathbb{Z}_N$ 轨形 sector 的凯勒模，γ 为对应 $\mathrm{SL}(2,\mathbb{Z})$ 模群的任意变换，轨形扭曲物质场的 p 重多重态 Φ_p 按照 [65, 66] 变换

$$\Phi_p \overset{\gamma}{\mapsto} (cT+d)^{n_p}\rho(\gamma)\,\Phi_p,\ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}). \tag{21}$$

Here $\rho(\gamma)$ is a representation of γ in a finite (double cover) modular group $\Gamma'_A = \mathrm{SL}(2,\mathbb{Z})/\Gamma(A)$ with A depending on the order N of the orbifold, and n_p is the (possibly fractional) modular weight carried by the twisted fields [54].

此处 $\rho(\gamma)$ 是 γ 在有限 (双覆盖) 模群 $\Gamma'_A = \mathrm{SL}(2,\mathbb{Z})/\Gamma(A)$ 中的表示，A 依赖于轨形的阶 N，n_p 是扭曲场携带的 (可能为分数的) 模权重 [54]。

The 4D effective supersymmetric field theory of such a model is governed by these symmetries. In particular, the modular transformation of the associated Kähler potential reads at leading order

这类模型的四维有效超对称场论由这些对称性支配，相关凯勒势的模变换在领头阶可以写为

$$K = -\ln\left(-\mathrm{i}T + \mathrm{i}\bar{T}\right) + \sum_p \left(-\mathrm{i}T + \mathrm{i}\bar{T}\right)^{n_p} \left|\Phi_p\right|^2. \tag{22}$$

This transformation is cancelled by a Kähler transformation of the superpotential provided that the superpotential terms of order m are given by

只要 m 阶超势项由下式给出，该变换就会被凯勒变换抵消

$$\mathcal{W} \supset Y(T)\,\Phi_{p_1}\cdots\Phi_{p_m} \tag{23}$$

and have total modular weight -1 per complex orbifold plane, where $Y(T)$ is a modular form.

且每个复轨形平面对应总模权重为-1，其中 $Y(T)$ 是模形式。

In the Narain formalism, the modular symmetries are identified with the subgroup of rotational outer automorphisms of the orbifold. Note that the R symmetries (cf. section " R Symmetries") may also be understood as remnants of the target-space modular transformations of the complex structure moduli of the orbifold. This implies a relation between the R charges of matter fields and their so-called modular weights [72].

在纳莱恩形式体系中，模对称性被等同于轨形旋转外自同构的子群。注意 R 对称性 (参见章节“R 对称性”) 也可以理解为轨形复结构模的靶空间模变换的剩余对称性，这意味着物质场的 R 荷与它们的所谓模权重之间存在关联 [72]。

Outer Automorphisms

外自同构

The effective action can exhibit certain outer-automorphism symmetries. These symmetries contain fundamental transformations like charge conjugation $\mathcal{C}$, parity $\mathcal{P}$, and time reversal $\mathcal{T}.\mathcal{CP}$ has to be broken in the flavor sector order to describe the real world, a criterion that already some of the first explicit string models turn out to satisfy [71]. Further outer automorphisms comprise the left-right parity of the left-right symmetric model, which may emerge as discrete remnants of the continuous gauge symmetries after orbifolding [16].

有效作用量可展现出特定的外自同构对称性。这些对称性包含电荷共轭 $\mathcal{C}$ 、宇称 $\mathcal{P}$ 这类基础变换，而时间反演 $\mathcal{T}.\mathcal{CP}$ 必须在味区破缺才能描述现实世界，这一判据实际上部分最早的显式弦模型就已经满足 [71]。其他外自同构包括左右对称模型的左右宇称，它可以是轨道紧致化后连续规范对称性遗留下来的离散对称性 [16]。

Note that in the Narain formalism some outer automorphisms of the orbifold can also be considered $\mathcal{CP}$ -like transformations. It remains to be seen whether there is a connection between such transformations and the physical $\mathcal{CP}$.

注意，在 Narain 形式体系中，轨道面的部分外自同构也可被视为 $\mathcal{CP}$ 类变换。这类变换与物理 $\mathcal{CP}$ 之间是否存在关联仍有待研究。

Approximate Symmetries and Hierarchies

近似对称性与层级结构

Starting at a symmetry-enhanced point has various benefits compared to analyzing generic points in moduli space. The models reviewed in this Chapter give rise to a variety of mildly broken, and thus approximate, symmetries. As already mentioned above, the latter may conceivably explain the observed hierarchies in the flavor sector [20]. They may also provide us with solutions to the μ and/or strong $\mathcal{CP}$ problems (cf. [25,59]). They may explain the scales in models of dynamical supersymmetry breaking (such as [56], which requires an explicit mass for some pairs of vector-like states) or the messengers of gauge mediated SUSY breaking [33].

与分析模空间的一般点相比，从对称性增强点出发具有诸多优势。本章评述的模型中存在大量轻微破缺的近似对称性。正如前文所述，这类对称性有望解释味 sector 中观测到的层级结构 [20]，也可为 μ 问题和/或强 $\mathcal{CP}$ 问题提供解决方案 (参见 [25,59])，还可解释动力学超对称破缺模型中的能标 (例如 [56]，该模型要求部分矢量型态对存在 explicit 质量)，或是规范介导超对称破缺的传递子能标 [33]。

Challenges

挑战

Modern days model building faces various challenges. Bottom-up models usually can accommodate observation but the shear abundance and flexibility of the emerging constructions make it appear unlikely that these activities alone will provide us with unique answers. This chapter focuses on top-down models, which come with their own challenges. They include:

当今的弦模型构建面临诸多挑战。自下而上的模型通常可以符合观测结果，但这类新兴结构种类繁多、灵活性极强，仅靠这类研究似乎不太可能为我们带来唯一的答案。本章聚焦自上而下的模型，这类模型也存在自身的挑战，包括:

C1. Obtain the correct gauge symmetry.

C1. 得到正确的规范对称性。

C2. Obtain the correct spectrum, i.e., the three generations of quarks and leptons without any other states which are chiral w.r.t. G_{SM} .

C2. 得到正确的谱，即三代夸克和轻子，不存在对 G_{SM} 手征的其他额外态。

C3. Avoid dangerous operators such as those leading to too fast proton decay or too large flavor changing neutral currents (FCNCs).

C3. 规避危险算符，例如导致质子衰变过快或味改变中性流 (FCNC) 过大的算符。

C4. Provide a consistent cosmological history.

C4. 给出自洽的宇宙学演化史。

C5. Reproduce the observed values of continuous parameters of the SM, i.e., the gauge and Yukawa couplings.

C5. 重现标准模型连续参数的观测值，即规范耦合和汤川耦合。

The first challenge, C1, has been mastered successfully in heterotic model building early on. Compactification breaks the gauge symmetry of the 10D heterotic string, and it is fairly straightforward to obtain the SM gauge symmetry, or a symmetry that can be broken to G_{SM} .

第一个挑战 C1，早在杂化弦模型构建中就已经成功解决。紧致化会破坏十维杂化弦的规范对称性，得到标准模型规范对称性，或是可以破缺到 G_{SM} 的对称性都相当直接。

Obtaining the correct spectrum, i.e., C2, has been a bigger challenge since string models may yield the wrong number of generations or give rise to chiral exotics. Nonetheless, extensive scans accompanied with appropriate search strategies have enabled the community to identify a large number of compactifications that exhibit the chiral spectrum of the SM at low energies (However, the number of chiral generations at low energies and at the compactification scale may be different [84], so some care needs to be taken not to prematurely discard models.). Notice that this often involves the appearance of extra, vector-like states which acquire masses below the compactification scale. While there are some constraints on such states, e.g., from the requirement that the gauge couplings remain perturbative, they also may play an important role in the phenomenology of the model, cf. our discussion in section "Chiral Zero Modes". Another concern stems from so-called fractionally charged exotics. As already mentioned in section "Early Universe," there are tight experimental constraints on their relic abundance (cf. e.g., [51,67]), so the appearance of such states in the spectrum leads to the requirement that they are not produced copiously in the early universe.

得到正确的谱，也就是 C2，一直是更大的挑战，因为弦模型可能得到错误的代数量，或是产生手征外态。尽管如此，配合合适搜索策略的大规模扫描已经让学界识别出大量在低能下具有标准模型手征谱的紧致化 (不过，低能和紧致化能标下的手征代数可能不同 [84]，因此需要注意不要过早排除模型)。需要注意的是，这通常会引入额外的矢量型态，它们会在紧致化能标以下获得质量。虽然这类态存在一些约束，例如要求规范耦合保持微扰性，但它们也可能在模型的唯象学中发挥重要作用，参见我们在“手征零模”一节的讨论。另一个问题来自所谓的分数电荷外态。正如“早期宇宙”一节已经提到的，实验对其残余丰度有严格约束 (例如参见 [51,67])，因此谱中出现这类态就要求它们不会在早期宇宙中被大量产生。

The offending operators mentioned in C3 include the so-called R -parity violating couplings of the MSSM, cf. our discussion below Equation (3). Forbidding this couplings requires additional symmetries, the simplest option being R - or matter parity. This shows, in particular, that it is not sufficient to obtain models with three generations and the SM gauge symmetry, one necessarily needs additional symmetries. The offending operators also often get induced by extra states. For instance, SU(3) triplet partners of the MSSM Higgs doublets may mediate proton decay at an unacceptably large rate [89]. This problem haunts 4D models of grand unification but is absent in certain higher-dimensional variants [3]. Nonetheless, it may get reintroduced through other vector-like states.

C3 中提到的危险算符包括最小超对称标准模型 (MSSM) 中所谓的破坏 R 宇称的耦合，参见我们对式 (3) 的讨论。禁戒这些耦合需要额外对称性，最简单的选择是 R 宇称或物质宇称。这尤其说明，仅得到具有三代和标准模型规范对称性的模型是不够的，还必须引入额外对称性。危险算符也通常会被额外态诱导产生。例如，MSSM 希格斯二重态的 SU(3) 三重态伙伴可能会引发速率大到不可接受的质子衰变 [89]。这个问题困扰着四维大统一模型，但在某些高维变体中不存在 [3]。尽管如此，它仍可能通过其他矢量型态重新出现。

While string cosmology is an active field (see Chap. 57, "Moduli Stabilization in String Theory" for more details), only limited attention has been given to the performance of otherwise promising models, which overcome C1 to C3. It remains a task for the future to see whether, say, inflation can be realized and a realistic baryon asymmetry can be generated.

虽然弦宇宙学是一个活跃领域 (更多细节参见第 57 章“弦论中的模稳定”)，但针对那些已经解决了 C1 到 C3、看起来很有前景的模型，人们对其相关性质的关注仍然有限。例如能否实现暴胀、能否产生现实的重子不对称性，这些都仍有待未来研究。

Challenge C5 is major and has not been completely mastered in any known construction so far, let alone in remotely realistic models. Part of the problem is that the couplings depend on the VEVs of certain scalar fields, the moduli (cf. section ”Moduli”), and possibly other fields (The precise definition of what a modulus is varies over the literature. In some parts D -flat combinations of the other fields are referred to as moduli.). This means that mastering challenge C2 requires stabilizing all moduli. This is a topic on its own, which is covered in Chap. 57, ”Moduli Stabilization in String Theory”. Within the examples given in section ”Examples,” we will comment on the extent to which realistic couplings are obtained.

挑战 C5 是最主要的，迄今为止在所有已知构造中都没有被完全解决，更不用说接近现实的模型了。部分问题在于耦合依赖于某些标量场即模的真空期望值 (参见“模”一节)，还可能依赖于其他场 (文献中对模的精确定义并不统一。在部分文献中，其他场的 D 平坦组合也被称为模)。这意味着，解决挑战 C2 需要稳定所有模，这本身就是一个独立课题，由第 57 章“弦论中的模稳定”专门讨论。在“实例”一节给出的例子中，我们会对得到现实耦合的程度进行评述。

Examples

示例

Geometric Orbifold with $\mathcal{N} = 1$ SUSY

具有 $\mathcal{N} = 1$ 超对称的几何轨形

Rather than reviewing extensive model scans, let us focus on a particular example, the model of [61]. The orbifold has noncontractible cycles (cf. section ”Classification of Toroidal Orbifold Geometries”) which allow one to break an SU(5) grand unified symmetry nonlocally down to G_{SM} . This type of GUT breaking avoids fractionally charged exotics. The spectrum consists of three chiral generations of quark and leptons plus additional states which are vector-like w.r.t. G_{SM} but massless at the orbifold point, see Table 1. Like the majority of models of this type, there is an FI term which has to be cancelled consistently with vanishing of the F - and (other) D-terms. The corresponding VEVs break the gauge symmetry at the orbifold point down to

我们不再综述大量的模型扫描，而是聚焦于文献 [61] 中的一个具体模型。该轨形存在不可收缩闭链 (参见“环面轨形几何分类”一节)，这些闭链允许我们将 SU(5) 大统一对称性非局域地破缺到 G_{SM} 。这种 GUT 破缺方式避免了带分数电荷的奇异态。谱包含三代夸克和轻子的手征世代，以及额外的相对于 G_{SM} 为矢量型但在轨形点处无质量的态，见表 1。和大多数此类模型一样，该模型存在一个 FI 项，该项必须在 F 项和其他 D 项都为零的前提下被一致抵消。对应的真空期望值将轨形点处的规范对称性破缺为

$$G_{\text{residual}} = G_{\text{SM}} \times \mathbb{Z}_4^R \times \text{SU}(2)_{\text{hid}}, \tag{24}$$

where G_{SM} and $\text{SU}(2)_{\text{hid}}$ stem from two different E_8 factors, and none of the SM matter is charged under $\text{SU}(2)_{\text{hid}}$.

其中 G_{SM} 和 $\text{SU}(2)_{\text{hid}}$ 来自两个不同的 E_8 因子，且所有标准模型物质都不携带 $\text{SU}(2)_{\text{hid}}$ 电荷。

These VEVs also provide mass terms for all SM charged exotics, yet the $\mathbb{Z}_4^R$ symmetry forbids the mass of one linear combination of Higgs fields, which gets identified with the MSSM Higgs pair. This pair acquires a mass after $\mathbb{Z}_4^R$ breaking. The order parameter of R symmetry breaking is the gravitino mass, i.e., of the order of the soft terms, which are assumed to be not too far above the electroweak scale. That is, the $\mathbb{Z}_4^R$, which is a discrete remnant of the Lorentz symmetry of compact space, can provide us with a solution to the μ problem along the lines of [7]. In addition, this $\mathbb{Z}_4^R$ suppresses dimension-5 proton decay operators enough to be consistent with observation.

这些真空期望值也为所有带标准模型电荷的奇异态提供了质量项，但 $\mathbb{Z}_4^R$ 对称性禁止了希格斯场一个线性组合获得质量，该组合被认定为最小超对称标准模型 (MSSM) 的希格斯对。该希格斯对在 $\mathbb{Z}_4^R$ 破缺后获得质量。R 对称性破缺的序参量是引力微子质量，即量级为软项的量级，一般假设软项不会比电弱标度高太多。也就是说，作为紧致空间洛伦兹对称性离散 remnant 的 $\mathbb{Z}_4^R$ ，可以沿着文献 [7] 的思路为我们提供 μ 问题的一个解决方案。此外，该 $\mathbb{Z}_4^R$ 可以充分压低 5 维质子衰变算符，使其与观测结果一致。

Table 1$\mathbb{Z}_4^R$ charges of the (a) matter fields and (b) Higgs and exotics. The index i in (a) takes values $i = 1, 2, 3$. The ℓ_i and h_i as well as the $\bar{d}_i$ and $\overline{\delta}_i$ are distinguished by their $\mathbb{Z}_4^R$ charges.

表中给出了 (a) 物质场和 (b) 希格斯与奇异态的 1$\mathbb{Z}_4^R$ 荷。(a) 中的指标 i 取值为 $i = 1, 2, 3$ 。ℓ_i 、h_i 以及 $\bar{d}_i$ 、$\overline{\delta}_i$ 通过它们的 $\mathbb{Z}_4^R$ 荷区分。

q_i	$\bar{u}_i$	$\bar{d}_i$	ℓ_i	$\bar{e}_i$	$\bar{h}_1$	h_2	h_3	h_4	h_5	h_6	$\bar{h}_1$	$\bar{h}_2$	$\bar{h}_3$	$\bar{h}_4$	$\bar{h}_5$	$\bar{h}_6$	δ_1	δ_2	δ_3	$\overline{\delta}_1$	$\overline{\delta}_2$	$\overline{\delta}_3$	
$\mathbb{Z}_4^R$	1	1	1	1	1	$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2	0	2	2	0	0

(a) Quarks and leptons (b) Higgs and exotics

(a) 夸克与轻子 (b) 希格斯与奇异态

It has been checked that qualitatively realistic fermion masses arise, i.e., the Yukawa couplings have full rank, exhibit hierarchies, and lead to nontrivial flavor mixing, and neutrino masses are see-saw suppressed. However, this is not to say that they are fully realistic, cf. our discussion of C5.

已经验证该模型可以得到定性现实的费米子质量，即汤川耦合满秩、存在层级性、产生非平庸味混合，且中微子质量通过跷跷板机制被压低。但这不意味着它们完全符合现实，参见我们对 C5 的讨论。

Altogether this example shows that explicit string models can successfully address some of the most pressing questions of (traditional) unified model building, including the μ and proton decay problems. However, it also illustrates that there is still a long way to go before we can claim to have found ”the” stringy SM. Apart from the question whether or not low-energy SUSY is realized in Nature, one has to successfully fix the moduli. While this is a topic on its own, which is covered in Chap. 57,"Moduli Stabilization in String Theory", the $\mathbb{Z}_4^R$ symmetry and charges can be used to show that generically there are no flat directions. States with odd $\mathbb{Z}_4^R$ acquire masses because the mass terms carry $\mathbb{Z}_4^R$ charge 2 (mod 4), and the fields of $\mathbb{Z}_4^R$ charge 2 pair up with linear combinations of $\mathbb{Z}_4^R$ charge 0 fields. Of course, generic statements do not always lead to the correct conclusions, and one has to verify explicitly that there are no flat directions, what the possible VEVs of the $\mathbb{Z}_4^R$ charge 0 fields are, and whether they lead to phenomenologically viable couplings in the SM sector.

该例总体表明，明晰的弦模型能够成功解答 (传统) 大统一模型构建中一些最紧迫的问题，包括 μ 问题和质子衰变问题。但它同时也说明，距离我们宣称找到“那个”弦标准模型还有很长的路要走。除了低能超对称是否在自然界中实现这一问题外，还需要成功固定模。尽管这本身就是一个独立课题，已在第 57 章“弦理论中的模稳定”中讨论，我们仍可利用 $\mathbb{Z}_4^R$ 对称性和荷证明，一般情况下不存在平坦方向。带有奇 $\mathbb{Z}_4^R$ 荷的态会获得质量，因为质量项携带 $\mathbb{Z}_4^R$ 荷 2(模 4)，且 $\mathbb{Z}_4^R$ 荷为 2 的场会与 $\mathbb{Z}_4^R$ 荷为 0 的场的线性组合配对。当然，一般结论并不总能得出正确结果，必须明确验证不存在平坦方向，确定 $\mathbb{Z}_4^R$ 荷为 0 的场可能的真空期望值是什么，以及它们是否能在标准模型领域产生符合唯象学的相互作用耦合。

Geometric Orbifolds Without SUSY

无超对称几何轨形

There is a consistent nonsupersymmetric heterotic string [5, 27, 29] , which can be understood as a freely acting $\mathbb{Z}_2$ orbifold of a $\mathcal{N} = 1$ heterotic string [27, 29] . Given the absence of evidence of supersymmetry at colliders, this version of the heterotic string deserves increasing attention, even though it does not exhibit the protection that SUSY offers against the appearance of tachyons, quadratic divergences, and a large cosmological constant [47].

存在一个自洽的非超对称杂化弦 [5, 27, 29] ，它可以被理解为 $\mathcal{N} = 1$ 杂化弦 [27, 29] 的自由作用 $\mathbb{Z}_2$ 轨形。由于对撞机上没有发现超对称存在的证据，这种杂化弦版本值得更多关注——尽管它不具备超对称对快子、二次发散和大宇宙常数的屏蔽作用 [47]。

The massless spectrum of this theory comprises three components: the gravitational part includes the graviton, the antisymmetric 2-form B_{MN} , and the dilaton; the gauge bosons of SO (16) × SO (16) arise in the gauge sector; and the charged matter states build the representations (128, 1) + (1, 128) + (16, 16).

该理论的无质量谱包含三个部分: 引力部分包含引力子、反对称 2-形式 B_{MN} 和 dilation；规范 sector 产生了 SO (16) × SO (16) 的规范玻色子；带电物质态构成表示 (128, 1) + (1, 128) + (16, 16)。

Applying similar compactification techniques such as orbifolds as in the super-symmetric case, there has been some effort to study the phenomenology of compactifications of this string theory in 4D, including models with a tachyon-free GUTs or SM massless spectrum [1, 2, 12, 77]. Although the progress does not yet compare to the supersymmetric case, some general features in SM-like models are known. In particular, the following properties of the massless spectrum are found: (i) at perturbative level, tachyons can be avoided; (ii) models with only one SM Higgs exist, but most of them exhibit a larger number of Higgs fields; (iii) there appear many fermion and scalar exotic states although there are models with a very small exotic spectrum; (iv) among the exotics, there are $\mathcal{O}$ (100) right-handed neutrinos; (v) leptoquark scalars are present in different amounts; and (vi) the number of fermions and bosons can coincide, yielding the possibility of an exponentially suppressed one-loop cosmological constant.

和超对称情形一样应用轨形这类紧致化技术后，已有不少工作研究该弦论四维紧致化的唯象学，包括无快子大统一理论或标准模型无质量谱的模型 [1, 2, 12, 77]。尽管目前进展还无法与超对称情形相比，人们已经了解类标准模型模型的一些普遍特征，具体发现无质量谱存在以下性质:(i) 微扰水平下可以避免快子；(ii) 存在仅含一个标准模型希格斯的模型，但多数模型的希格斯场数量更多；(iii) 总会出现大量费米子和标量奇异态，不过也存在奇异谱极轻的模型；(iv) 奇异态中包含 $\mathcal{O}(100)$ 右手中微子；(v) 存在不同数量的轻夸克标量；(vi) 费米子和玻色子的数量可以相等，因此单圈宇宙常数可以被指数压低。

As an example, let us focus on the model 2 of [77], based on an Abelian orbifold compactification of the $\mathcal{N} = 0$ string (in the bosonic formulation). It includes the SM gauge group and additional SU(2) and U(1) factors. In the fermionic sector, there are only three SM generations arising from twisted sectors and 119 right-handed neutrinos. In the scalar sector, besides 9 Higgs doublets and 9 scalar leptoquarks, there are 30 SM singlets, which may be considered flavons of a traditional $\Delta(54)$ flavor symmetry. The modular flavor features of this kind of models are not known.

举个例子，我们来看文献 [77] 中的模型 2，它基于 $\mathcal{N} = 0$ 弦 (玻色表述下) 的阿贝尔轨形紧致化。该模型包含标准规范群，以及额外的 SU(2) 和 U(1) 因子。费米子 sector 中，只有三代来自扭转 sector 的标准模型费米子代，以及 119 个右手中微子。标量 sector 中，除了 9 个希格斯二重态和 9 个标量轻夸克，还有 30 个标准模型单态，可以被视为传统 $\Delta(54)$ 味对称性的味微子。这类模型的模味性质目前还不清楚。

Smooth Compactifications

光滑紧化

As already mentioned, one can obtain smooth compactifications of the heterotic $E_8 \times E_8$ theory in ten dimensions. If the compactification is to preserve $\mathcal{N} = 1$ supersymmetry in four dimensions, the compact space has to be a CY manifold [24]. Models with the chiral spectrum of the MSSM have been found in this approach, see, e.g., [10]. Notice that, a priori, it is not clear that every supergravity compactification of this type has a stringy origin [48] but is expected that a substantial fraction of the models in the literature correspond to string models. Machine learning techniques have been utilized to efficiently find models with the gauge symmetry and chiral spectrum of the SM [86]. It will be interesting to see if the absence of certain terms [6, 88] can be understood in terms of ordinary symmetries as is the case in the orbifold models discussed here, or if novel mechanisms are at play. In the latter case, this may provide us with new ways of overcoming C3. In passing, let us mention that a significant amount of smooth models can be obtained from orbifolds via blow-up (cf. e.g., [46]). In particular, giving VEVs to fields that are massless at the orbifold point often amounts to resolving the orbifold singularities. A detailed discussion of these interesting topics is, however, beyond the scope of this chapter.

如前文所述，我们可以得到十维杂化 $E_8 \times E_8$ 理论的光滑紧化。若要求紧化在四维时空保留 $\mathcal{N} = 1$ 超对称，则紧化空间必须是 CY 流形 [24]。该方法已经找到了具备最小超对称标准模型手征谱的模型，参见例如文献 [10]。需要注意的是，从先验上看，并不是所有这类超引力紧化都一定存在弦论起源 [48]，但文献中的大部分模型都预期对应弦论模型。研究者已经利用机器学习技术高效搜寻具备标准模型规范对称性和手征谱的模型 [86]。某些项的缺失 [6, 88] 能否像本文讨论的轨形模型那样用普通对称性解释，还是存在新的作用机制，这一问题值得研究。如果是后者，它可能会给我们提供克服 C3 的新方法。顺便一提，大量光滑模型可以通过轨形的吹胀得到 (参见例如文献 [46]): 给轨形不动点处无质量的场赋予真空期望值，通常就等价于解决轨形奇点。但限于本章范围，我们不对这些有趣课题展开详细讨论。

Where Do We Stand?

我们目前进展如何?

The aim of heterotic model building is to reproduce and interpret particle physics in the heterotic string. This can be achieved by identifying appropriate compact-ifications. As we have discussed, various approaches have led to large sets of semi-realistic models that exhibit the matter spectrum of the standard model, its minimal supersymmetric version, as well as certain gauge extensions such as GUTs. Using various techniques, the effective symmetries of these constructions have been studied, which has yielded interesting implications for flavor physics, $\mathcal{CP}$ violation, proton stability, supersymmetry breaking, among other features.

杂合模型构建的目标是在杂合弦中复现并诠释粒子物理，这可以通过确定合适的紧致化实现。如我们此前讨论，多种方法已经得到了大量半现实模型集合，这些模型具备标准模型、最小超对称标准模型的物质谱，也包含大统一理论这类规范扩展模型。利用各类方法，我们已经研究了这些构造的有效对称性，得到了其对味物理、$\mathcal{CP}$ 破坏、质子稳定性、超对称破缺等诸多性质的重要启示。

However, a clear, let alone unique, picture has not yet emerged. The gauge and Yukawa couplings are, in principle, consistent with observation in a subset of the models. However, solid and precise predictions of the latter have remained largely elusive so far. This is hardly surprising. To see why, recall that we believe to know the Lagrange density of QCD in great detail but it remains a challenge to precisely compute basic quantities like the proton mass. In string phenomenology, the analogous analyses are even more challenging as the computation of many observables requires, among other things, a precise, quantitative understanding of moduli stabilization, which has not yet obtained. However, one can turn this around by saying that the explicit models provide us with a framework in which progress in these open questions can lead to testable predictions for the many parameters of the SM as well as BSM physics. We expect that this framework will also deliver a picture to address some of the pressing puzzles in cosmology.

但目前尚未出现清晰的图景，更不用说唯一确定的框架了。原则上，在部分模型中，规范耦合和汤川耦合与观测结果一致。但到目前为止，我们一直没能得到汤川耦合可靠精准的预言。这其实并不意外: 我们都知道，我们已经对量子色动力学的拉格朗日密度有了非常细致的了解，但精确计算质子质量这类基础量仍是难题；而在弦唯象学中，类似的分析难度还要更高——计算很多可观测量都需要对模稳定有精准定量的理解，这一点目前还未实现。不过换个角度看，这些显式模型为我们提供了一个框架: 随着这些开放性问题取得进展，我们就能得到标准模型众多参数以及超出标准模型物理的可检验预言。我们也期待这个框架能为解决宇宙学中若干紧迫难题提供思路。

Cross-References

交叉引用

- A Lightning Introduction to String Theory
  - 弦论简明导论
- Moduli Stabilization in String Theory
  - 弦论中的模稳定

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